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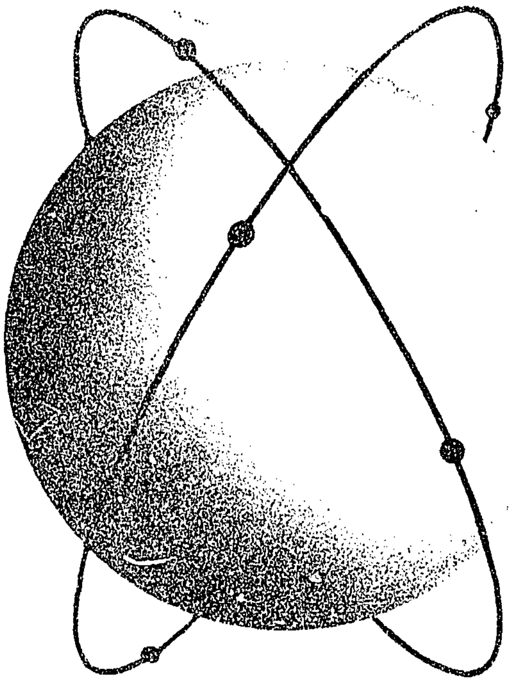
ABSTRACT

As the sixth lesson of the Articulated Multimedia Physics Course, instructional materials are presented in this study guide with relation to the uniformly accelerated motion of bodies starting from rest. The objective is to teach students how a complete set of equations of motion is derived and how to use them. Free falling bodies near the Earth's surface are also discussed. The content is arranged in scrambled form, and the use of matrix transparencies is required for students to control their learning activities. Students are asked to use magnetic tape playback, instructional tapes, and single concept films at the appropriate place in conjunction with the worksheet. Included are a problem assignment sheet and a study guide slipsheet. Related documents are SE 015 963 through SE 015 977. (CC)

ARTICULATED MULTIMEDIA PHYSICS



LESSON 6

NEW YORK INSTITUTE OF TECHNOLOGY

OLD WESTBURY, NEW YORK

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Old Westbury, Long Island

New York, N.Y.

ARTICULATED MULTIMEDIA PHYSICS

Lesson Number 6

UNIFORMLY ACCELERATED MOTION OF BODIES STARTING FROM REST

IMPORTANT: Your attention is again called to the fact that this is not an ordinary book. It's pages are scrambled in such a way that it cannot be read or studied by turning the pages in the ordinary sequence. To serve properly as the guiding element in the Articulated Multimedia Physics Course, this Study Guide must be used in conjunction with a Program Control equipped with the appropriate matrix transparency for this Lesson. In addition, every Lesson requires the availability of a magnetic tape playback and the appropriate cartridge of instructional tape to be used, as signaled by the Study Guide, in conjunction with the Worksheets that appear in the blue appendix section at the end of the book. Many of the lesson Study Guides also call for viewing a single concept film at an indicated place in the work. These films are individually viewed by the student using a special projector and screen; arrangements are made and instructions are given for synchronizing the tape playback and the film in each case.

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New York Institute of Technology

Articulated Multimedia Physics

LESSON 6

STUDY GUIDE SLIPSHEET

STUDY GUIDE TEXT: Page 39, near center of page. Change 9.80 mi/sec^2 to 9.80 m/sec^2. This is the correct value of gravitational acceleration in the mks system. "mi" is a typographical error.

STUDY GUIDE DIAGRAMS: No corrections.

WORKSHEETS: Page 177, Worksheet for Tape Segment 2, Data Item A.

Note that the complete statement of the relation given in this Data Item is

$$F_e = \frac{kq_1q_2}{r^2}$$

in which F_e is the electrical force between particles, q_1 and q_2 are the charges on the particles, and r is the separation distance between them. If one deals with the same two particles throughout a given problem, then q_1 and q_2 do not change (remain constant). Thus, the charges and the constant k may be combined in the form of a quantity that is still a constant. This is the form used in the Data Item.

$$F_e = \frac{k}{r^2}$$

This form is all you need to answer the Worksheet questions. Please add the subscript "e" under the "F" in the worksheet equation.

HOMEWORK PROBLEMS: No corrections.

If you have recently completed Lesson 5 of this series Graphs of Motion, you may be puzzled by the title of this lesson. You ask, "Haven't we already studied uniformly accelerated motion of bodies starting from rest?"

The subject of motion has more than one aspect. We have closely scrutinized distance, speed, and time from the point of view of graphical interrelationships. We have seen that a v-t graph for a uniformly accelerating body is a straight line, that the area under the curve represents the distance the body travels in the stipulated time, that the actual acceleration of the moving object may be obtained from the slope ($\Delta v/\Delta t$) of the v-t curve, and that acceleration is measured in units like mi/hr-sec, km/hr-sec, and ft/sec^2. All of this will be very helpful in studying the new material in this lesson because you have gained a clear mental picture from the graphs of the ways in which distance, speed, and time are related to each other.

Now, what aspect of motion concerns us in this lesson? The processes of simple algebra may be easily adapted to the solution of motion problems. While a graph is superb for clarifying an idea or a relationship, it is sometimes inconvenient and often time-consuming to draw one to scale. On the other hand, algebraic solutions of motion problems, while they do not draw pictures for you, do offer a speedy and accurate technique that has great value for the student of physics.

Please go on to page 2.

Of course, there was also some algebra involved in the study of graphs. Since we shall want the simple little equations already developed in your notebooks in consolidated form, let's start the notes for this lesson with the following entry:

NOTEBOOK ENTRY

Lesson 6

1. Review of Basic Equations of Motion

   (a) For a body moving with constant speed.
   (1) $d = vt$ (2) $v = d/t$ (3) $t = d/v$

   (b) Average speed of a body moving with random speed changes.
   $\bar{v} = d/t$

   (c) Average speed of a body moving with uniformly changing speed and starting from rest.
   $\bar{v} = v/2$ where v = final velocity.

   (d) The slope of a d-t curve for a body moving with uniform speed gives the speed of the body.
   (d-t) slope $= v = \Delta d/\Delta t$.

   (e) The slope of a v-t curve for a body moving with uniform acceleration gives the acceleration of the body.
   (v-t) slope $= a = \Delta v/\Delta t$.

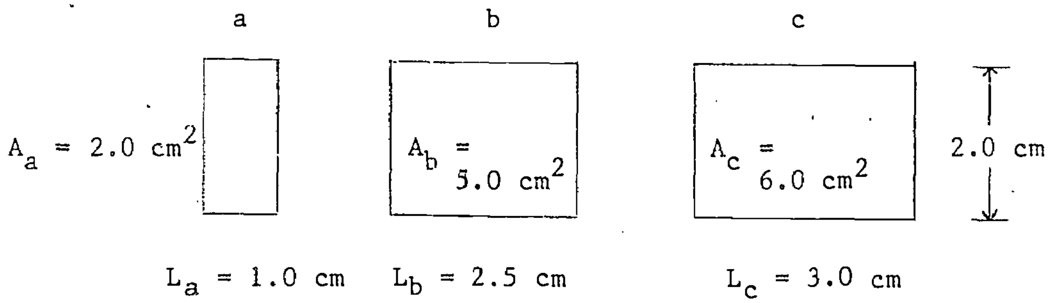
Our objective in this lesson is to teach you how a complete set of equations of motion is derived and how to use these equations once you have learned them. We shall also show that bodies in free fall near the Earth's surface follow the same set of equations, making it possible to predict many interesting things about falling bodies.

Please go on to page 3.

Just as a precautionary measure, we are going to start this lesson with a review of <u>proportions</u>, even though we know that you studied this subject in elementary arithmetic and again in algebra. The physicist makes frequent use of proportions, however, and we'd best be sure that you have a good working knowledge of them.

Proceed now to the start of the lesson by turning to page 4.

To get you started on proportions, we'll consider a very easy kind of relationship involving the areas of rectangles.



$$A = L \times W$$

Figure 1

Figure 1 shows three rectangles having different lengths but identical widths. Since the area (A) of a rectangle is always $A = L \times W$ (L = length, W = width), we can summarize the information about these rectangles this way:

$$A_a = L_a \times W = 1.0 \text{ cm} \times 2.0 \text{ cm} = 2.0 \text{ cm}^2$$

$$A_b = L_b \times W = 2.5 \text{ cm} \times 2.0 \text{ cm} = 5.0 \text{ cm}^2$$

$$A_c = L_c \times W = 3.0 \text{ cm} \times 2.0 \text{ cm} = 6.0 \text{ cm}^2$$

From this we can see that if the width <u>is held constant</u>, then multiplying the length by 2.5 (2.5 cm is 2.5 times as large as 1.0 cm) causes the area to increase by a factor of 2.5 (5.0 cm^2 is 2.5 times as large as 2.0 cm^2). Or, multiplying the length by 3 (3.0 cm is 3 times as large as the original 1.0 cm) causes the area to increase by the same factor since 6.0 cm^2 is just 3 times as large as 2.0 cm^2.

Quantities like the length and area in this example are said to be <u>directly proportional</u> to each other. By whatever factor you multiply (or divide) the length, the area increases (or decreases) by exactly the same factor.

In our example above, if you were to increase the length to 8 cm (this is multiplying the initial length by a factor of 8), how many times as large as the original area would the new area be?

(1)

A 8 times as large.

B 16 times as large.

This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

YOUR ANSWER --- B

This is not the simplest possible form of the definition of acceleration for a body that starts from rest and accelerates uniformly

You apparently realized that Δv reduces to $\underline{v}$ because you replaced the Δv by just plain $\underline{v}$ in your answer. But a similar thing happens to the Δt term, doesn't it?

$$\Delta t = t - 0 \text{ where } \underline{t} \text{ is the final time}$$

Why, then, did you leave the denominator Δt instead of simplifying it?

Please return to page 150. You can find a form simpler than $a = \Delta v/\Delta t$.

YOUR ANSWER --- B

You probably weren't careful in your arithmetic. It appears you recognized that the distance is proportional to the square of the time and that the second interval is 4 times as long as the first (i.e. 8 sec = 4 x 2 sec).

Since $d = kt^2$, then:

For 2 sec 28 m = $k(t)^2$ where t = 2 sec.
For 8 sec ?? m = $k(4t)^2$ where 4t = 8 sec.
?? m = $k \times 16t^2$

Since the right side of the last equation is 16 times as large as the right side of the first equation, then the left side of the latter must also be 16 times as large as the left side of the former. But the left side of the first equation is 28 m; hence the left side of the last equation must be 16 x 28 m.

You apparently obtained <u>8</u> as the square of 4. That would explain why your answer came out just half as large as it should.

Please return to page 138 and select the correct answer.

YOUR ANSWER --- A

You are correct. You filled in the words distance and velocity.

Well, let's see how such an equation is obtained. We shall first write the two fundamental equations we already have for bodies that start from rest and accelerate uniformly.

$$(1) \quad a = v/t$$
$$(2) \quad d = \tfrac{1}{2}at^2$$

From what has been said above, you can see that we want an equation that contains a, d, and v. The t terms should not appear in it. That is, we want to eliminate the t terms from the two equations.

The easiest way to do this is to first solve equation (1) for t and then substitute this new value of t in equation (2). Thus:

$$\text{from (1):} \quad t = v/a$$

We then substitute v/a in place of t in equation (2) this way:

$$d = \frac{a \times (v/a)^2}{2} = \frac{a \times v^2/a^2}{2}$$

$$d = \frac{v^2/a}{2} = \frac{v^2}{2a}$$

It appears, then, that we should be able to find the height of rise of a vertically projected object (or the distance fallen by an object that is dropped from a height) by squaring the final velocity and dividing by twice the acceleration.

Please go on to page 10.

Our mathematics has given us this equation for relating acceleration, distance, and velocity:

$$d = v^2/2a$$

Before accepting this relation, we should perform one more operation. We have done this throughout our work; do you remember what it is?

Let's see. After deriving a general equation in physics, we should insure its validity by which of the following?

(39)

A Testing it on a new problem.

B Working it in reverse to see if we come out with the same expression with shich we began.

C Performing a dimensional or unit check on it.

YOUR ANSWER --- A

On the face of it, this answer is much, much too large. A bomb moving with a speed of 640,000 ft/sec would be moving at 43,700 miles per hour! Even high-speed orbital satellites don't travel at this speed!

We know what caused this error. You forgot the square-root sign.

Please return to page 60 and try again.

YOUR ANSWER --- A

Your arithmetic is apparently correct, but your algebra isn't. This answer is not acceptable.

For the case of a body brought to rest from some initial speed, we can no longer use the simple expression $a = v/t$. In order to keep our algebraic signs correct, we must resort to the general form of the equation: $a = \Delta v/\Delta t$.

We have proved that the acceleration of a body that is slowing down is a negative quantity; we have also shown that its change of velocity is negative as well.

Then, transposing Δt and $\underline{a}$ in the equation above, we have

$$\Delta t = \frac{\Delta v}{a} = \frac{v_2 - v_1}{a} = \frac{0 \text{ ft/sec} - 88 \text{ ft/sec}}{-15 \text{ ft/sec}^2}$$

$$\Delta t = \frac{-88 \text{ ft/sec}}{-15 \text{ ft/sec}^2} = ?$$

There is a (-) sign in the numerator; there is a (-) in the denominator. Algebraically speaking, what is the sign of the quotient? Knowing this, you can see why the answer you chose is wrong.

Please return to page 26 and select the alternative answer.

YOUR ANSWER --- A

You are correct. When two symbols like I_1 and R_1 are placed adjacent to one another as in I_1R_1, you assume the presence of a "times" sign.

Thus, I_1R_1 is the product of I_1 and R_1. Also, I_2R_2 is the product of I_2 and R_2. So, when you see $I_1R_1 = I_2R_2$, you can read it as: "The product of I_1 and R_1 equals the product of I_2 and R_2."

O.K. If the products are equal to one another, then which of the following statements is also true as a consequence of this?

(9)

A If numerals are substituted for the symbols, then I_1R_1 will have an answer different than I_2R_2.

B If numerals are substituted for the symbols, then I_1R_1 will have the same answer as I_2R_2.

YOUR ANSWER --- B

You're not thinking this through.

You want to determine distance, d. The expression ($a = v/t$) does not contain a distance term, so how can you solve it for distance?

Thus in selecting a relation that will help you answer a question from the real world, you must be sure that the equation contains all the terms you are given plus the quantity you are to find.

Please return to page 114. You should be able to select the correct answer this time.

YOUR ANSWER --- A

This sounds good but isn't! Seconds are numbered as shown below:

```
0 ---- 1 sec ---- 2 sec ---- 3 sec ---- 4 sec ---- 5 sec ---- 6 sec
 first    second     third    fourth     fifth     sixth
  sec      sec        sec      sec        sec       sec
```

So, if you subtract the distance fallen in 5 sec from the distance fallen in 6 sec, you have found the distance fallen <u>during</u> which second? The sixth, of course. This gives the answer away, doesn't it?

Please return to page 126 and select the correct answer.

YOUR ANSWER --- B

You are correct. Since a = v/t, then:

$$a = \frac{24 \text{ ft/sec}}{3 \text{ sec}} = 8\frac{\text{ft}}{\text{sec}^2}$$

Very often we meet situations in which the acceleration of the body is known and we have to determine its final velocity, knowing its time of travel. Here is a sample problem of this type.

A body in free-fall near the surface of the Earth accelerates uniformly at the rate of about 9.80 meters per second per second. If it were allowed to fall for 2.00 sec, what would be its final velocity? (From now on, unless told otherwise, you may assume uniform acceleration and a start from rest condition.)

(16)

A Its final velocity would be 4.90 m/sec.

B Its final velocity would be 19.6 m/sec.

YOUR ANSWER --- B

This answer doesn't agree with our definition of k as a constant that does not change under any circumstances.

If two quantities are proportional, then multiplying one of them by some fraction will cause the other to be multiplied by the same fraction. Now, B is directly proportional to A because that is what is meant by B = kA. Hence, this rule must apply.

When we say that A is reduced by 1/3, we mean that the new A is 1/3 as large as the old A. What then must happen to B?

Please return to page 132 and select another answer.

CORRECT ANSWER: If the distance is increased 5 times, the intensity is reduced to 1/25 of its former value. That is,

$$\text{Initially:} \qquad I = \frac{B}{d^2}$$

$$\text{After a fivefold increase:} \qquad I = \frac{B}{(5d)^2} = \frac{B}{25d^2}$$

The right-hand term in the new equation is 1/25 the initial value; hence the intensity is reduced to 1/25 of its former value.

NOTEBOOK ENTRY

Lesson 6

(Item 2)

(e) The expression $y = kx^2$ states that y is directly proportional to the square of x.

(f) The expression $y = k/x^2$ states that y is inversely proportional to the square of x.

For the remainder of this lesson we shall be concerned with developing and using the equations of motion for a very specific kind of motion which we can describe as follows:

BODIES THAT START FROM REST AND ACCELERATE UNIFORMLY.

Which of the graphs in Figure 4 describes this kind of motion?

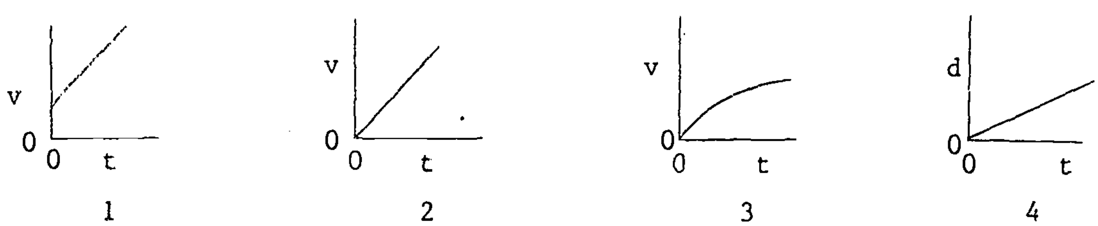


Figure 4

(12)

A Graph 1 B Graph 2 C Graph 3 D Graph 4

YOUR ANSWER --- A

More than that! We hope that this error was due to carelessness rather than lack of understanding.

If distance were directly proportional to time, then the proportion would read $d = kt$. But that isn't it, is it?

Please be more careful.

Please return to page 164 and select another answer.

YOUR ANSWER --- C

Here's Group 3 again:

$$W = kf \qquad\qquad E/W = k$$

You'd be able to recognize the type of proportion implicit in $W = kf$ if you would manipulate the expression so that <u>k</u> would be alone on one side of the equation. In this way you could see immediately whether the <u>ratio</u> or the <u>product</u> of the variables was equal to a constant. The conversion we suggest is most easily accomplished by dividing both sides of $W = kf$ by f. This gives:

$$W/f = k$$

This is a constant ratio. So, what kind of proportion is it?

Now $E/W = k$ is already in its correct form for analysis. Again we have a ratio of variables equal to a constant.

Since both proportions are direct, of course, your answer is not correct.

Please return to page 146 and select another answer.

YOUR ANSWER --- C

This answer is incorrect.

The equation for determining distance is $d = \frac{1}{2}at^2$. You substituted in the numerator properly but forgot to divide by 2. Review the problem and correct your error.

Please return to page 139 and select the correct answer.

CORRECT ANSWER: It takes 40 sec for the flour sack to fall to Earth.

If your answer checks with ours, turn to page 23.

If your answer is different from ours, return to page 117 and reconsider the problem. Work it out again until your answer is $t = 40$ sec.

Now turn to page 23, please.

Have you ever wondered how high you can throw a ball straight up in the air? If you know the initial velocity you can impart to the ball, you can now calculate the height to which it will rise. With the information now in your possession, you have to handle this problem in two steps. You will first have to determine the time required for the ball to reach the peak of its rise, and then you can calculate the height to which it rose. (Later we shall derive one more general equation that will permit you to find the height in a single step.)

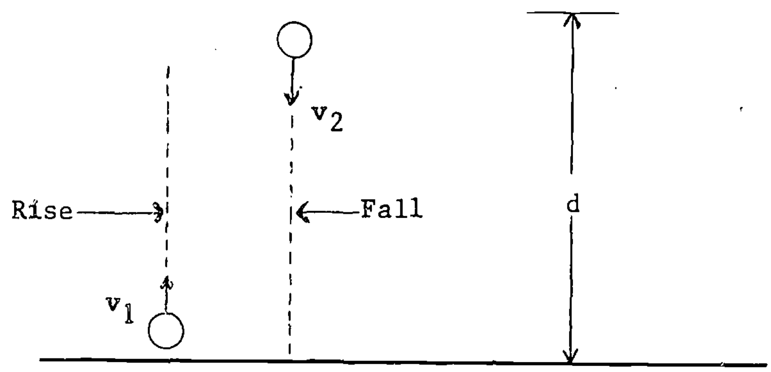


Figure 7

Refer to Figure 7. We shall consider the rise of the ball to the highest point in its ascent. Imagine that you are standing on the ground and that you throw the ball straight up. If the throw is perfect, the ball will come down along exactly the same path. In our drawing, we have displaced the fall path slightly to the right only to prevent confusion.

We have shown two velocities, v_1 and v_2. Considering only the rise path, what do these velocities represent?

(34)

A v_1 is final velocity and v_2 is initial velocity.

B v_1 is initial and v_2 is final velocity.

C Both v_1 and v_2 are initial velocities.

YOUR ANSWER --- A

This is not correct. For the following set of values:

$$\frac{A_a}{L_a} = 2.0 \text{ cm} \quad \text{and} \quad \frac{A_b}{L_b} = 2.0 \text{ cm} \quad \text{and} \quad \frac{A_c}{L_c} = 2.0 \text{ cm}$$

we find that each of these operations give a constant number, 2.0 cm. We might have expected this, of course, since dividing area by length is bound to give the width of the rectangle, and we know this is constant since we made it so to begin with.

But you said that the two quantities are proportional if their product is a constant. Certainly the operations shown above are not products, are they?

Please return to page 52 and select the correct answer.

YOUR ANSWER --- B

You are correct. Here's the proper format and solution:

Given	Find	Equation	Substitutions
$a = 7.00\ m/sec^2$	d	$d = \frac{1}{2}at^2$	$d = \dfrac{7.00\frac{m}{sec^2} \times (3.80\ sec)^2}{2}$
$t = 3.80$ sec			

$$d = \frac{7.00\frac{m}{\cancel{sec^2}} \times 14.44\ \cancel{sec^2}}{2}$$

$d = \underline{50.5}\ \underline{m}$ (answer)

So, the cart starting from rest and picking up speed as it rolls down an inclined plane will travel a distance of 50.5 meters in the allotted time of 3.80 sec.

Another very practical type of situation with which automotive engineers must often wrestle involves the effectiveness of the brakes of a newly designed car. A very important question is this: "If the rate of deceleration (slowing down or negative acceleration) and the initial velocity of a car are known, how long will it take the brakes to bring the car to rest?"

We shall pause a moment here to refresh your memory. Our original definition of acceleration, you will recall, is $a = \Delta v/\Delta t$. We simplified this by omitting the "Δ's" only for the condition of a body starting at rest and accelerating uniformly. Now, the conditions have changed. This car starts with a given velocity and then comes to rest. In all our previous examples, the final velocity was a larger number than the initial velocity because the body gained speed; now, the final velocity will be a smaller number than the initial velocity because the body is decelerating.

Please go on to page 26.

Where previously v_2 (end of trip speed) was larger than v_1 (start of trip speed), the reverse will now be true. What will be the effect of this on our equations?

We'll use some figures to help clarify this. Suppose a car is moving at a speed of 30 mi/hr (or 44 ft/sec). The brakes are applied, causing the car to come to rest in 2.2 sec. Let us find the deceleration in ft/sec^2. Using the general equation:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t} = \frac{0 \text{ ft/sec} - 44 \text{ ft/sec}}{2.2 \text{ sec}} = \frac{-44 \text{ ft/sec}}{2.2 \text{ sec}}$$

$$a = -20 \text{ ft/sec}^2$$

Note that deceleration is, therefore, negative acceleration and is indicated by a (-) sign before the quantity. Note also that Δv, the change in velocity, is also a negative quantity because the final speed is less than the initial speed. Thus, in solving any deceleration problem in which the body is brought to rest uniformly, remember that a is negative and that Δv is also negative.

Right? Now let's see how long it takes to bring to rest a car moving at a speed of 60 mi/hr initially (88 ft/sec) whose brakes are capable of producing a deceleration of 15 ft/sec^2. Use the general equation and find t. What answer do you get? Select one of these:

(23)

A t = -5.9 sec.

B t = 5.9 sec.

CORRECT ANSWER: I_3 must take on the value of 1.

If R_3 has become 16, then I_3 must take on the value of 1 in order than the product of I_3R_3 will still be equal to 16 as the expression dictates.

Thus it should now be clear to you that the expression:

$I_1R_1 = I_2R_2 = I_3R_3$ ... and so on may be restated this way:

$$IR = k \text{ (a constant)}$$

Now, go back to item 2(d) in your notes. Keep your finger on it while we return to the original question.

Please return to page 71.

YOUR ANSWER --- B

Right. But the original question did not ask for this information, did it?

To return to the original question, please turn to page 126.

YOUR ANSWER --- A

This statement is not incorrect. To see why, first determine which relationship is implied in the statement. Since it involves a, d, and t, we will write:

$$d = \tfrac{1}{2}at^2$$

If acceleration (a) is constant, then a/2 is constant and may be replaced by k:

$$d = kt^2$$

Transposing to get the k alone on one side, we move the t^2 to the left and into the denominator:

$$\frac{d}{t^2} = k$$

Thus, we have a constant ratio signifying a direct proportion; this means that d is directly proportional to t^2 or, in words, the distance is directly proportional to the square of the time if the acceleration is held constant.

Please return to page 78. Try once more to locate the incorrect statement.

YOUR ANSWER --- A

Not right! Is your notebook in order?

Please turn to page 31 and make this entry now!

NOTEBOOK ENTRY

Lesson 6

1. Review of Basic Equations of Motion

(a) For a body moving with constant speed.

$$(1)\quad d = vt \qquad (2)\quad v = d/t \qquad (3)\quad t = d/v$$

(b) Average speed of a body moving with random speed changes.

$$\bar{v} = d/t$$

(c) Average speed of a body moving with uniformly changing speed and starting from rest.

$$\bar{v} = v/2 \text{ where } v = \text{final velocity.}$$

(d) The slope of a d-t curve for a body moving with uniform speed gives the speed of the body.

$$(d\text{-}t)\text{ slope} = v = \Delta d/\Delta t$$

(e) The slope of a v-t curve for a body moving with uniform acceleration gives the acceleration of the body.

$$(v\text{-}t)\text{ slope} = a = \Delta v/\Delta t$$

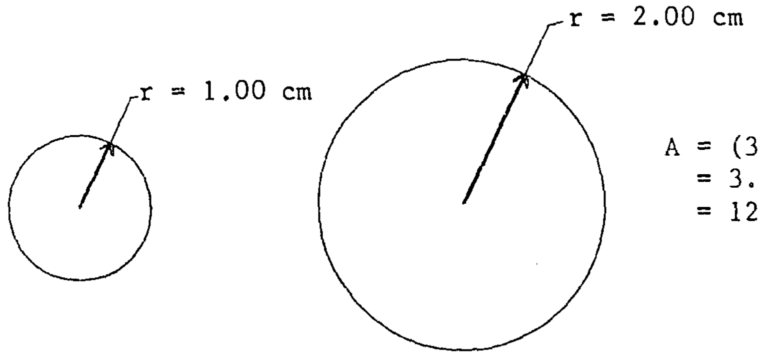
Please go on to page 120.

YOUR ANSWER --- B

You are correct. Since π never changes, it may be considered to be the constant of proportionality.

Hence, in words, $A = kr^2$, reads as follows: "The area of a circle is directly proportional to the square of the radius."

$$A = (3.14)(1)^2 = 3.14 \text{ cm}^2$$



$$A = (3.14)(2)^2 = 3.14 \times 4 = 12.6 \text{ cm}^2$$

Figure 2

To extract the full meaning from this statement, refer to Figure 2. Circle 1 has a radius of 1.00 cm; hence an area of 3.14 cm^2 as indicated. Circle 2 has double the radius and an area of 12.6 cm^2. Since 12.6 is 4 times as great as 3.14 (to 3 significant figures, of course), we see at once that: If the radius of a circle is doubled, its area increases 4 times.

We could have obtained this answer directly from the proportionality much more easily. First we write $A = kr^2$. Next we ask, "What will happen to the area A if the radius is doubled?" To answer this, we write: $A = k(2r)^2$. Now, this next point is extremely important. To show the doubled radius, we wrote 2r, but this entire quantity, (2r), must be placed inside the parentheses to show that the whole thing is to be squared. So, squaring we have: $A = k(4r^2) = 4kr^2$. The quantity on the right is now 4 times as large as it was initially; hence the area must have increased by a factor of 4.

Please turn to page 33 to continue.

In the last step, we doubled the radius and obtained a fourfold increase in area. What happens to the area if we triple the radius as compared to its initial length of 1.00 cm?

(5)

A It will increase by a factor of 3.

B It will increase by a factor of 6.

C It will increase by a factor of 9.

YOUR ANSWER --- A

You are correct. We know a and t, and we want to find d. All these terms, and no others, are contained in the relation $d = \frac{1}{2}at^2$.

CAUTION: Don't expect to always have an equation handy. You may have to combine several or find a quantity for one equation by using another.

Returning to our car problem, we will find it very helpful to list the given quantities in terms of their symbols like this:

Given	Find	Appropriate Equation
$a = 10\ ft/sec^2$ $t = 4\ sec$	d	$d = \frac{1}{2}at^2$

Substitutions

$$d = \frac{10\frac{ft}{sec^2} \times (4\ sec)^2}{2} = \frac{10\frac{ft}{\cancel{sec^2}} \times 16\ \cancel{sec^2}}{2} = \underline{80\ ft}$$

Now you try one. A cart rolling down an inclined plane has a uniform acceleration of $7.00\ m/sec^2$. How many meters will it roll if it starts from rest and travels for 3.80 sec?

FOLLOW THE PROBLEM FORMAT ABOVE. Write what is given in terms of the symbols; indicate what is to be found; select the appropriate equation containing all the terms you can use; then substitute, carrying units, and solve. Only one of the answers below is correct. Which is it?

(22)

A 13.3 m

B 50.5 m

C 101 m

YOUR ANSWER --- C

We believe we know what you were thinking when you selected this answer.

Certainly, v_1 is the initial velocity for the rise path because the entire action starts when you impart this velocity to the ball. Also, v_2 is certainly the initial velocity of the ball as it starts downward on the fall path.

But, we asked you to consider the rise path only. On its upward flight, the ball is being decelerated by the gravitational force. When it reaches the highest point in its rise path, it must come to rest before it can reverse its direction and begin to fall.

Then v_2 may be considered either the end of the rise path or the beginning of the fall path. But if we restrict our discussion to the rise path, then we must think of v_2 as what? Is it an initial or a final velocity?

Please return to page 23. Choose the correct answer this time.

YOUR ANSWER --- D

You are correct. Group 2 is: $vt = k$, and $P = k/V$.

The relation $vt = k$ is a clear-cut inverse proportion because it contains a constant product. The other one, $P = k/V$ is not as easy to see right away, but by transposing V to the left side, we obtain $PV = k$. Here, again, is a constant product or an inverse proportion.

There is one other type of relationship that appears in physics with considerable frequency. We think that you can figure it out for yourself without further explanation. Suppose you try. Don't let the meanings of the symbols distract you from the main idea of interpreting the relationship.

Please go on to page 37.

The amount of illumination that falls on a screen from a given source of light depends upon the brightness of the source and upon the distance between the source of light and the screen. The equation that relates them is:

$$I = B/d^2$$

where I = intensity of illumination, B = brightness of light source, and d = the distance between screen and source.

Now picture this: We keep the brightness (B) constant by working with the same lamp; then we gradually increase the distance (d). Naturally, the illumination (I) will decrease as the screen gets farther away. With B constant, which one of the following statement correctly describes the relationship?

(11)

A The intensity of illumination is not proportional in any way to the distance between screen and source.

B The intensity of illumination is inversely proportional to the distance between screen and source.

C The intensity of illumination is inversely proportional to the square of the distance between screen and source.

D The intensity of illumination is directly proportional to the distance between screen and source.

- YOUR ANSWER --- C

You are correct. The formal procedure would go something like this:

$$d = kt^2$$

First interval $28 \text{ m} = kt^2$ where $t = 2$ sec.
Second interval $?? \text{ m} = k(4t)^2$ where $4t = 8$ sec.
$?? \text{ m} = 16kt^2$
Hence $448 \text{ m} = 16kt^2$ since <u>448</u> = 16 x 28.

NOTEBOOK ENTRY

Lesson 6

Item 3(b)

(1) Since a/2 is constant, the proportionality may be written as:

$$d = kt^2 \text{ where } k = a/2$$

This shows that the distance traveled by a body starting from rest and accelerating uniformly is directly proportional to the square of the time of travel.

As a final check on your understanding of the relationship between time of travel and distance covered, suppose you answer this question. A uniformly accelerating body starting from rest covers a distance d in a certain time interval t. If the same body is allowed to accelerate for a time interval 9 times as long as in the first instance, what will be the distance it will cover?

(29)

A 9d

B 81d

YOUR ANSWER --- B

You are correct. The solution follows:

Given	Find	Equation	Substitutions
$t = 3$ sec $a = g = 32$ ft/sec^2	d	$d = \frac{at^2}{2}$	$d = \frac{32 \text{ ft/sec}^2 \times 9 \text{ sec}^2}{2}$ $d = 16 \times 9 \text{ ft/sec}^2 \times \text{sec}^2$ $d = \underline{144}\ \underline{\text{ft}}$

Try a similar problem in the mks system.

A stone is thrown straight up into the air with an initial velocity of 24.5 m/sec. Taking $g = 9.80$ m/sec^2, find:

(a) the time required for the stone to <u>return</u> <u>to</u> <u>Earth</u> after the instant of throwing.

(b) the distance to which the stone rose above the Earth at the top of its ascent.

Work this out to three significant figures. Then choose the group below in which both answers are correct.

(37)

A <u>Group</u> <u>4</u>
$t = 5.00$ sec
$d = 61.2$ m

B <u>Group</u> <u>3</u>
$t = 5.00$ sec
$d = 30.6$ m

C <u>Group</u> <u>2</u>
$t = 3.10$ sec
$d = 30.6$ m

D <u>Group</u> <u>1</u>
$t = 2.50$ sec
$d = 100$ m

YOUR ANSWER --- A

Maybe you got your new and old terms mixed up.

If two quantities are directly proportional, then multiplying one of them by some fraction will cause the other to be multiplied by the same fraction.

When we say that A is reduced by 1/3, we mean that the new A is 1/3 as large as the old A. Then what must happen to B?

Please return to page 132 and select another answer.

YOUR ANSWER --- B

You filled in the words distance and time. We already have such a relationship. Doesn't the expression

$$d = \frac{1}{2}at^2$$

contain the terms? Doesn't it give the relationship between acceleration, distance, and time? Certainly it does.

So please return to page 172 and pick a better answer.

YOUR ANSWER --- D

Very bad guess! Is your notebook in order?

Go to page 31 and make this entry now.

YOUR ANSWER --- C

All right, we'll run through it on one or two other equations.

Say we want to check d = vt (for uniform motion). Since mks units are used, d is measured in meters, v is measured in meters per second, and t is measured in seconds. We substitute these units in the equation:

$$d = vt$$

$$m = m/sec \times sec$$

Next, we perform whatever indicated operations are possible, handling the unit abbreviations as though they were numbers. Notice on the right side that we have sec in the denominator of the first term and sec in the numerator of the second term; hence we can cancel sec against each other.

$$m = m/\cancel{sec} \times \cancel{sec}$$

This leaves us with: $m = m$

The dimensional check is considered successful if both sides of the unit equation turn out to be the same.

To do a unit check on $v^2 = 2ad$, use m/sec for v (which gives you m^2/sec^2 for v^2), m/sec^2 for a, and m for d.

Please try to run through it before you proceed. Then return to page 127 and select the "yes" answer.

YOUR ANSWER --- D

Not right! You're not taking the time to analyze the equation in terms of the product or ratio idea. We have e[illegible]ized this before: if in doubt, transpose terms until the constant fa[illegible]r is on one side of the equation by itself. Then see whether there is a constant ratio or a constant product.

In this example, the B is constant (the same lamp is moved closer to or farther from the screen). To get B alone on one side, the d^2 must be transposed. Multiplying both sides by d^2 gives us:

$$I \times d^2 = \frac{B \times d^2}{d^2}$$

$$\text{So} \quad Id^2 = B.$$

What condition does this indicate? Is there a constant product or a constant ratio? You now have all the clues.

Please return to page 37 and select another answer which fits all the conditions.

YOUR ANSWER --- C

This answer has a reasonable ring; it is of the correct order of magnitude. Nevertheless, it is off.

You did not use the 2 under the radical. That is, $v = \sqrt{\underline{2}ad}$.

So, please return to page 60. Rework the problem and then choose the right answer.

YOUR ANSWER --- A

No. Go back and review your notes on proportions. You seem to have forgotten the common sense behind these mathematical symbols. If a factor on one side of the equation $d = kt^2$ should increase, the factor on the other side will have to increase also. Such a relation is a direct proportion. Changes are in the same direction.

Please return to page 101 and select the alternative answer.

YOUR ANSWER --- B

You are correct. The basic expression, a = v/t, should first be solved for the unknown, v, to obtain: v = at. Upon subsititution, we find that:

$$v = 9.80 \text{ m/sec}^2 \times 2 \text{ sec} = 19.6 \text{ m/sec}$$

Using the same reasoning and remembering that the acceleration of a freely falling body near the Earth's surface is approximately 9.80 m/sec^2, work out the following problem.

How long would it take a freely falling body near the Earth's surface to acquire a velocity of 78.4 m/sec?

Work out the answer in these steps: (1) solve the literal equation a = v/t for t as the unknown; (2) substitute the known numbers and units; (3) express the answer with the correct units. Write it all out; then check it against our solution.

Please turn to page 124.

YOUR ANSWER --- A

The answer is incorrect.

You did not square the time as called for in $d = \frac{1}{2}at^2$.

Please return to page 139. Rework the problem and then select the best answer.

YOUR ANSWER --- C

You'll have to be more careful in deciding which term is to be the constant of proportionality.

In this case, π is the constant because it is an invariant number having the value 3.14. On the other hand, the radius is a <u>variable</u> whose value you can change at will; of course, when the radius is changed, the area must also change.

In changing an equation into a proportionality, you must remember that the variables (like radius and area) are not altered in any way. All you need do is to recognize that you can substitute <u>k</u> for the one factor that is constant.

Please return to page 77 and select another answer.

YOUR ANSWER --- C

No, your units are incorrect. The foot is a unit of distance, not acceleration. You may have come out with this answer because you handled the units improperly while substituting in the equation.

The right way to do this is shown below:

$$a = \frac{v}{t} = \frac{24\ \text{ft/sec}}{3\ \text{sec}}$$

In this expression, the seconds in the denominator do not cancel the seconds in the numerator as you may have thought. If you find difficulty in handling units in fractional form as above, then simplify the expression by the usual process of converting the division to multiplication by the reciprocal of the denominator. That is, the reciprocal of 3 sec is 1/3 sec. This gives the following:

$$a = 24\frac{\text{ft}}{\text{sec}} \times \frac{1}{3\ \text{sec}}$$

So you can see that the "sec" units do not cancel out.

Please return to page 110; pick the answer that takes these units into account properly.

YOUR ANSWER --- C

You apparently do not recognize the proportionality present in this expression because it is stated in a form that is different from the ones we have used before.

Take the first two terms only: $I_1R_1 = I_2R_2$. Let's see what this means.

It states simply that the product I_1R_1 is equal to the <u>same number</u> as the product I_2R_2. Saying this another way, as the quantity R (whatever it may be) changes from a value R_1 to R_2, the quantity I (whatever <u>it</u> may symbolize) changes from I_1 to I_2 <u>in such a way as to keep the product the same</u>. That is, as variations in R cause corresponding variations in I, the product of the corresponding pairs always gives the same answer. In other words, <u>the product of I and R is a constant</u>.

Refresh your memory by going back to your notes. Does a constant product indicate a proportionality at all? If it does, what kind of proportion is indicated?

Please return to page 71 and select another answer.

YOUR ANSWER --- A

You are correct. That is,

$$A_a = L_a \times W = 1.0 \text{ cm} \times 2.0 \text{ cm} = 2.0 \text{ cm}^2, \text{ and}$$

$$A_{new} = L_{new} \times W = 8.0 \text{ cm} \times 2.0 \text{ cm} = 16.0 \text{ cm}^2$$

Since 16.0 cm^2 is 8 times as large as 2.0 cm^2, the area does increase by a factor of 8.

We can make this a more general statement in this fashion: Two quantities are in direct proportion if multiplying or dividing one of them by any factor a causes the other to be similarly multiplied or divided by the same factor a.

Refer to Figure 1 on page 4 while investigating the following relationships:

$$\frac{A_a}{L_a} = \frac{2.0}{1.0} = \underline{2.0} \ \underline{\text{cm}} \qquad \frac{A_b}{L_b} = \frac{5.0}{2.5} = \underline{2.0} \ \underline{\text{cm}} \qquad \frac{A_c}{L_c} = \frac{6.0}{3.0} = \underline{2.0} \ \underline{\text{cm}}$$

Note that each ratio gives the same answer. This happens only when the two quantities are directly proportional to each other. Thus, in the future we will know that two quantities are directly proportional if which of the following is true?

(2)

A Their product is a constant.

B Their ratio is a constant.

YOUR ANSWER --- A

No, that's not true.

Why isn't it? Well, we can start by realizing that if $\overline{v} = d/t$ and the same $\overline{v} = v/2$, then we ought to be able to set the two right-hand members equal to each other because things equal to the same thing are equal to each other. Thus:

$$\frac{d}{t} = \frac{v}{2}$$

This much is certainly correct. Now, in the expression you chose as the valid one, the equality above is solved for _d_ as the unknown.

To get _d_ alone on the left side of the equation, the _t_ must be eliminated from that side. We can do this by multiplying both sides of the equation by _t_. _Try this for yourself._ You will find that you do not get $d = 2vt$ as a result of this operation. What do you get? This is the answer to the original question, of course.

Please return to page 120 and select another answer.

YOUR ANSWER --- B

You are doing something wrong. Try it once more, using the units m/sec for _v_ so that the units for $\underline{v^2}$ are m^2/sec^2; use m/sec^2 for _a_, and _m_ for _d_.

Then return to page 127 and select the "yes" answer. This will carry you to the page where the dimensional check is worked out and you can compare it with yours.

CORRECT ANSWER: No, of course not! There are tall men with thin necks and short men with thick ones. There is no systematic variation of neck size with height: nobody can derive an equation to relate these; hence there can be no proportionality.

However, when an equation relates two quantities, there must be some kind of proportionality between them because changing one must inevitably result in a perfectly predictable change in the other.

Now please return to page 37 and make another choice.

YOUR ANSWER --- B

That's not correct. You did not square the time. Remember, the equation is:

$$d = \frac{at^2}{2}$$

Since the time of fall is 5 sec, then you must substitute 25 sec^2 for $\underline{t^2}$.

Please return to page 149 after recalculating the distance; then select a better answer.

YOUR ANSWER --- C

This answer group is not entirely correct.

At this point we feel we should not tell you the source of your error. We'd prefer that you rework the problem, paying careful attention to all details.

Please return to page 39. Do the problem again and select the right answer.

YOUR ANSWER --- A

$$f/m = k \qquad\qquad pv = k$$

The first of these expressions states that the ratio of f to m is a constant. In other words, whatever value we give to variable m, then f will change in such a way as to keep the ratio f/m a constant. Thus if we double m, f will have to double also to maintain a constant quotient; if we triple m, f will have to triple; and so on. This has all the earmarks of what kind of proportion? Right! It's a direct proportion.

Now we'll look at pv = k. Here, the product of two variables (p and v) is constant. This means that if we double v, then p will have to be halved if the product is to remain unchanged. Double one, halve the other; triple one, divide the other by three; quadruple one, divide the other by four, etc. One goes up, the other goes down in the same proportion. What is this? An inverse proportion, of course.

So we see that Group 1 contains a direct proportion and an inverse proportion; hence the answer you chose cannot be correct.

Please return to page 146 and select another answer.

YOUR ANSWER --- B

You've made a rather common error. In squaring 3, you got 6. Is 6 the square of 3? No, it isn't.

Please return to page 33 and select another answer.

CORRECT SOLUTION:

Given	Find	Equation
$v = 96.0$ ft/sec $a = g = 32.0$ ft/sec^2	d	$d = \frac{v^2}{2a}$

Substitutions

$$d = \frac{(96.0\ \text{ft/sec})^2}{2 \times 32.0\ \text{ft/sec}^2} = \frac{9216\ \text{ft}^2/\text{sec}^2}{64\ \text{ft/sec}^2} = \underline{144}\ \underline{\text{ft}}.$$

Consider this problem: a stationary balloon at an altitude of 1.00×10^4 ft drops a bomb. Assume no air resistance. With what velocity will the bomb strike the Earth?

You will notice immediately that time is not given. This means that since we know $a = g = 32.0$ ft/sec^2 and we know that altitude = distance = 1.00×10^4 ft, we shall want to use the equation that relates v, d, and a. Since we want to find v, we shall then substitute directly in $v = \sqrt{2ad}$.

Set up the problem in the approved format and solve it. Note, too, that we have expressed the altitude of the balloon in scientific notation in order to let you know that the quantity is correct to three significant figures. Select the answer below that corresponds to yours.

(41)

A The impact velocity of the bomb will be 640,000 ft/sec.

B The impact velocity of the bomb will be 800 ft/sec.

C The impact velocity of the bomb will be 566 ft/sec.

YOUR ANSWER --- A

Sorry! You must have this relation turned around. What you are saying here is that the velocity acquired by a uniformly accelerating body starting from rest is directly proportional to the square of the distance.

We have seen that $v = \sqrt{2ad}$, when constant acceleration, 2a, is a constant replaceable by k. This means that:

$$v = k\sqrt{d} \quad \text{or} \quad \frac{v}{\sqrt{d}} = k$$

So v is directly proportional to the square root of d. This means that if d is multiplied by a factor of 6, then v will be multiplied by a factor of $\sqrt{6}$.

Please return to page 122 and select a better answer.

YOUR ANSWER --- A

You're working too fast. Take it easy!

The substitution looks like this:

$d = vt/2$ to start with.

Then, $d = (at)t/2$ when we substitute <u>at</u> for <u>v</u>.

The numerator contains two time symbols to be multiplied together. Now look at the answer you chose. You see that it has omitted one of the "t's"; hence it must be incorrect.

Please turn to page 141 and select the other answer.

This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

YOUR ANSWER --- C

Sorry! You have forgotten an important characteristic of uniform acceleration in graphical representations. A curved-line graph of the v-t form such as Graph 3 in Figure 4 on page 18 does not show uniform acceleration. That is, the body is not picking up the same amount of speed during each equal interval of time.

If it were, what shape would the curve have?

Please return to page 18 and look over the other curves carefully before making another choice.

YOUR ANSWER --- B

You are confusing the <u>actual</u> area of the new rectangle with the <u>ratio</u> of the new to original area.

The original length was 1.0 cm and, with a width of 2.0 cm, the original area is 2.0 cm^2. That is,

$$A_a = L_a \times W = 2.0 \text{ cm} \times 2.0 \text{ cm} = \underline{2.0 \text{ cm}^2}$$

Now, if we increase the length by a factor of 8, the new length is 8 cm and we have:

$$A_{new} = L_{new} \times W = 8.0 \text{ cm} \times 2.0 \text{ cm} = \underline{16.0 \text{ cm}^2}$$

So the new area is 16 cm^2, and this is probably the reason you chose the answer above. But the question was, "How many times as large as the original area would the new area be?"

How many times as large as 2.0 cm^2 is 16.0 cm^2? This is the way to think out this answer.

Please return to page 4 and select the correct answer.

YOUR ANSWER --- C

You've forgotten how to recognize a direct proportion.

Get k alone on one side of the equation by transposing. To do this here, transpose t^2 to the other side where it goes into the denominator. Thus:

$$\frac{d}{t^2} = k$$

This shows that d and t^2 are in constant ratio, not constant product. Hence, the implied proportionality is not inverse. Notice what happens to d if t^2 is increased.

Please return to page 164. You should have no trouble selecting the correct answer now.

YOUR ANSWER --- C

You are correct. The fifth second is the interval between the end of second number 4 and the end of second number 5.

```
0 ---- 1 sec ---- 2 sec ---- 3 sec ---- 4 sec ---- 5 sec ---- 6 sec
  first     second      third     fourth      fifth      sixth
   sec        sec        sec        sec        sec        sec
```

All right. Find the distance fallen by a stone dropped from the top of the 550-ft building during its fifth second of fall. Work this out on scrap paper. Check it carefully.

Then turn to page 88 and compare your answer with ours.

YOUR ANSWER --- B

You are correct. At the top of the rise path, the ball comes to a full stop so that its final velocity, v_2, is zero. (If we think of the fall path at all, we would then consider v_2 as the initial velocity for this action. Of course, we would then treat the falling ball as a body starting from rest since $v_2 = 0$.)

You should see now that the rise of an object thrown straight upward is a motion that strongly resembles that of an automobile being braked to a stop. The deceleration (or negative acceleration) is uniform and is assigned a (-) sign; the object is slowing down so that Δv is also (-).

All right. Suppose you gave the ball an initial velocity of 96.0 ft/sec as you threw it. Remembering that $a = g = 32.0 \text{ ft/sec}^2$, you could now calculate the time required for the ball to reach the highest point in its ascent.

Using the relation $a = \Delta v/\Delta t$, which one of the following presents the facts correctly?

(35)

A $t = \dfrac{96.0 \text{ ft/sec}}{32.0 \text{ ft/sec}^2}$

B $t = \dfrac{-96.0 \text{ ft/sec}}{-32.0 \text{ ft/sec}^2}$

YOUR ANSWER --- C

You filled in the words velocity and time.

We already have an equation that relates acceleration, velocity, and time, don't we? The relationship is:

$$a = v/t$$

So this isn't it.

Please return to page 172. Find the right answer.

YOUR ANSWER --- A

You are correct. Using D = M/V, we have:

$$D = \frac{96\ g}{200\ cm^3} = 0.48\ g/cm^3.$$

So we see that when the volume is doubled, mass remaining the same, the density decreases to half its former value. This is exactly what we mean by an inverse proportion.

NOTEBOOK ENTRY

Lesson 6

(Item 2)

(c) Two quantities are inversely proportional to each other if multiplying one of them by a given factor causes the other to be divided by the same factor, or vice versa.

(d) Two quantities are inversely proportional if their product is a constant.

Item (d) is easily proved for our rubber block example. If M is held constant, we may substitute k, the proportionality constant, for mass. Thus: D = k/V. Multiplying both sides of the equation by V, we obtain DV = k. Thus, the product of the two variables is a constant.

Testing with the numbers:

$$D_1V_1 = 0.96\ g/cm^3 \times 100\ cm^3 = 96.0\ g$$

$$D_2V_2 = 0.48\ g/cm^3 \times 200\ cm^3 = 96.0\ g$$

Make this part of your mental stock in trade. If 2 variables have a constant ratio, they are directly proportional; if they have a constant product, they are inversely proportional.

Please go on to page 71.

In the study of electricity, you will meet something like the following: $I_1R_1 = I_2R_2 = I_3R_3 \ldots$, and so on. Does this expression indicate a proportionality? Choose one of the answers below.

(7)

A I and R are directly proportional.

B I and R are inversely proportional.

C I and R are not proportional at all.

D I don't know what it indicates.

YOUR ANSWER --- D

Although this equation does contain distance and time terms, the term for acceleration is missing; hence you need something more or a different equation.

In selecting a relation that will answer a specific question or solve a specific problem, you must be sure that the equation contains not only terms that fit the quantities given, but also a term containing the unknown.

Please return to page 114 and choose a better answer.

YOUR ANSWER --- A

What happened to the square-root relationship?

You answered this question as though v were directly proportional to d (if a is constant). This is not true.

You know that v is directly proportional to the *square root of* d. Make use of this fact in arriving at your answer.

Please return to page 173. Be sure you know why you must select the other answer.

YOUR ANSWER --- B

You're ignoring the fact that the velocity acquired by a uniformly accelerating body starting from rest is directly proportional to the square root of the distance that the body travels.

Your answer would be right if $v = kd$. But it doesn't!

You know that $v = k\sqrt{d}$ or, in ratio form, $v/\sqrt{d} = k$. Thus, the acquired velocity is directly proportional to the square root of the distance. So if d is multiplied by a factor of 6, then v will be multiplied by a factor of $\sqrt{6}$.

Please return to page 122. You should be able to recognize the right answer now.

YOUR ANSWER --- B

That's not right. Earlier in this lesson we emphasized and re-emphasized that two quantities are proportional to each other if they are related by a constant as in $d = kt^2$.

Furthermore, if a series of corresponding values for _d_ and _t^2_ are substituted for the symbols in the equation, either their _ratio_ or their _product_ will come out as a constant number, depending on the kind of proportion represented by the equation.

In this case, when the equation is solved for _k_, that is,

$$\frac{d}{t^2} = k$$

we see that it predicts a constant _ratio_. Thus, a proportion must exist between _d_ and _t^2_.

Please return to page 148 and select the other answer.

YOUR ANSWER --- C

You are correct. You recognized from $B = kA$ that B is directly proportional to A and that in this situation whatever is done to A must also be done to B.

The principle of proportionality is nicely applicable to the simple equations of motion we have studied. For example, consider a body moving with constant speed, $d = vt$. To make this equation conform with the ideas we have just discussed, we can substitute k in place of v since the speed is specified as constant.

$$d = kt$$

In words, this expression should sound like this: "The distance covered by a body moving with constant speed is directly proportional to the time of the trip." Here, v is the constant of proportionality. So if you double the time, the distance traveled will double; if you quadruple the time, the distance will quadruple.

In the same way, if you want to drive farther in a given length of time, you will have to drive faster. That is, if time is held constant,

$$d = kv$$

If in normal driving you can make 99 miles in three hours, your average speed is 33 miles per hour. If you wish to make 150 miles in the same time, you will have to average 50 m. p. h. To make 300 miles in three hours, you have to average 100 m. p. h. Better fly!

Please go on to page 77.

Going a little further, we next consider how to handle proportionalities of the <u>second degree</u>. We term an expression like $d = vt$ an equation of the first degree because it has no square or square root signs in it. But the area of a circle is a second degree equation:

$$A = \pi r^2 \qquad (\pi = 3.14)$$

Now, π is an invariant number; it never changes its value. Thus, which of the following correctly converts the equation for the area of a circle into a proportionality?

(4)

A $A = kr$

B $A = kr^2$

C $A = \pi k^2$

YOUR ANSWER --- B

You are correct. The solution follows:

Given	Find	Equation
$d = 10^4$ ft $a = g = 32.0$ ft/sec^2	final velocity (v)	$v = \sqrt{2ad}$

Substitutions

$$v = \sqrt{2 \times 32.0 \text{ ft/sec}^2 \times 10^4 \text{ ft}} = \sqrt{64.0 \times 10^4 \text{ ft}^2/\text{sec}^2} = 8.00 \times 10^2 \text{ ft/sec}$$

$$= \underline{800 \text{ ft/sec}}$$

Before testing you on this lesson, we want to run through some of the proportionalities implicit in the three equations:

(1) $a = v/t$ (2) $d = \frac{1}{2}at^2$ (3) $v = \sqrt{2ad}$

One of the statements below is incorrect. Pick it out by selecting the indicated letter. (All of these assume zero initial velocity and uniform acceleration.)

(42)

A If acceleration is constant, then distance is directly proportional to the square of the time.

B If the time of travel for several bodies is constant, then the final velocity of a body is directly proportional to the acceleration.

C If acceleration is constant, then the final velocity of a body is directly proportional to the distance the body travels.

D If the time of travel for several bodies is held constant, then the distance covered is directly proportional to the acceleration.

YOUR ANSWER --- A

This answer group is not entirely correct.

Rather than tell you where your error or errors are, we'd prefer that you go back and rework the problem, giving careful attention to all details.

Please return to page 39. After solving the problem again, you should be able to choose the right answer.

YOUR ANSWER --- C

You have not selected the simplest possible form. You saw that, for this case, Δt reduces to just plain $\underline{t}$, so you substituted $\underline{t}$ for Δt in the original relation.

But a similar thing happens to the Δv term, doesn't it?

$$\Delta v = v - 0 \text{ where } \underline{v} \text{ is the final velocity}$$

Why, then, did you leave the numerator Δv instead of simplifying it?

Please return to page 150. You can find a form simpler than $a = \Delta v/t$.

CORRECT SOLUTION:

(1) $a = v/t$; thus $v = at$.

(2) $v = 32 \text{ ft/sec}^2 \times 4 \text{ sec} = 128 \text{ ft/sec}$.

NOTEBOOK ENTRY

Lesson 6

Item 3(a)

(1) In mks units, the acceleration produced by the force of gravity near the surface of the Earth is approximately 9.80 m/sec/sec (symbolized by g).

(2) In English units, the acceleration produced by the force of gravity near the surface of the Earth is approximately 32 ft/sec/sec (symbolized by g).

Before continuing, please turn to page 178 in the blue appendix.

Notebook Check

Refer to notebook entry 1, Lesson 6.

Which two items in this entry present relationships in which average speed is specifically mentioned? Write the two relationships on a piece of scrap paper. DON'T PENALIZE YOURSELF BY GUESSING AT THESE. CHECK YOUR NOTEBOOK. Select the correct choice of the two items as identified by their sub-item letters below.

(17)

A. Items (a) and (b).

B. Items (b) and (c).

C. Items (d) and (e).

D. Items (a) and (e).

YOUR ANSWER --- B

You are correct. If both sides of the equation are divided by t^2, we get $d/t^2 = k$. Thus, the quantities d and t^2 are in constant ratio, signifying a direct proportion between them.

Going further, if the distance covered by a uniformly accelerating body starting from rest is directly proportional to the square of its travelling time, then we should be able to make some predictions about distances covered in real life situations.

Suppose that we have a slightly inclined trough down which a marble, starting from rest anywh re along its length, will roll with gradually increasing speed. The ough is marked at 3 inches from the bottom, and at 12 inches from the bottom. We can adjust the angle of the trough so that when the marble is placed at the 3-inch mark, it requires 1 second to reach the bottom. At this angle, how many seconds do you think the marble would require to reach the bottom from the 12-inch mark?

2 seconds

3 seconds

4 seconds

Select what you think would be the best answer and then turn to page 109.

YOUR ANSWER --- A

How would this insure its validity? If the problem is a new one, we don't know what the answer should be, so how can we tell if the equation is correct? We might test the new equation on an old problem whose answer we already know if we happen to have such a problem handy. Then if the new answer agrees with the old one, we are pretty certain that the new equation is correct.

There is a much more satisfying way to do this without the need of testing it on a new problem. Try to remember how we did this before.

Please return to page 10 and choose a better answer.

YOUR ANSWER --- C

Somehow, you inverted the definition of density.

The equation is: $D = M/V$ where M = mass in grams and V = volume in cubic centimeters. In this example, the volume is a larger quantity than the mass (numerically); hence the density D must turn out to be less than unity. Your answer is larger than 1; consequently you must have divided volume by mass instead of doing it correctly the other way around.

Please return to page 168 and choose a better answer.

YOUR ANSWER --- C

This answer is a real boner!

You want to determine the <u>distance</u>, <u>d</u>. The expression $\overline{v} = v/2$ contains no distance term, nor does it contain a time term, nor does it contain an acceleration term! In other words, it contains nothing that will help you.

In selecting a relation that will answer a question or solve a problem, you must be sure this relation contains not only terms that fit the quantities given, but also a term containing the quantity you are to find.

Please return to page 114 and find the equation you really can use.

YOUR ANSWER --- B

You are correct. The expression $k = d/t^2$ tells you that the quantities d and t^2 are in constant ratio. Any quantities in constant ratio are directly proportional to each other.

Our original dilemma was that we obtained the following experimental data:

Trip length (d)	Time (t)
3 inches	1 second
12 inches	2 seconds

and we had to use this to find out whether or not these figures proved that distance is directly proportional to the square of the time.

The method that works out best in most cases is to substitute pairs of corresponding values in the ratio d/t^2 and see whether you obtain the same number for each ratio. Let's do it.

$$\text{First: } \frac{d}{t^2} = \frac{3}{(1)^2} = \frac{3}{1} = 3$$

$$\text{Second: } \frac{d}{t^2} = \frac{12}{(2)^2} = \frac{12}{4} = 3$$

Is this proof enough? You may now return to the original question; you should be able to handle it.

Please return to page 109 and select the correct answer.

YOUR ANSWER --- A

Think carefully. The body starts from rest at the top of the building. This is the zero-distance point of reference. Then when you apply $d = \frac{1}{2}at^2$, you determine the distance downward that the stone has dropped from this zero reference point. Thus, application of $d = \frac{1}{2}at^2$ to this problem or to any problem involving simple free-fall, provides a figure that states the distance the body has dropped below the original zero-distance.

Now, if you subtract this from the height of the building, you no longer have the distance fallen, but rather the distance that is still left to fall.

Please return to page 97 and choose the alternative answer.

CORRECT SOLUTION:

For t = 4 sec, $d = \frac{32 \text{ ft}/\cancel{\text{sec}^2} \times 16 \cancel{\text{sec}^2}}{2} = 256$ ft.

For t = 5 sec, $d = \frac{32 \text{ ft}/\cancel{\text{sec}^2} \times 25 \cancel{\text{sec}^2}}{2} = 400$ ft.

Distance fallen during fifth second = 400 ft - 256 ft = 144 ft. (Note: Building height has no effect on solution, unless you want to find the distance fallen during the sixth second.)

For a variation in problem type, we present the one below. This will give you practice in literal transformation of equations as well as some work in mks units.

A balloon, hovering motionless at a height of 7.84 km drops a floursack marker which falls to Earth. Assuming zero air friction and uniform acceleration, how long does it take the sack to make impact?

Clearly, since we know a = g = 9.8 m/sec^2 and d = 7.84 km or 7,840 m, we shall be using the relation below to solve this problem.

$$d = \frac{at^2}{2}$$

But for what unknown should this equation be solved in its literal form before substitutions are made?

Decide on your answer; then turn to page 161 to verify it.

YOUR ANSWER --- D

All right. We'll have to "dig" into the substance of this a bit deeper.

Referring to the expression $I_1R_1 = I_2R_2$, which one of the following statements is true?

(8)

A I_1 times R_1 equals I_2 times R_2.

B I_1 divided by R_1 equals I_2 divided by R_2.

YOUR ANSWER --- C

Not right! Is your notebook in good order?

Go on to page 31 and make the entry now.

YOUR ANSWER --- A

Remember what the conditions are. We are considering the rise path only. You stand on the ground and throw the ball upward with a velocity of v_1. When the ball reaches the highest point of its ascent and is just ready to start falling downward, it has the velocity v_2.

On this basis, the answer you selected contradicts the facts, doesn't it?

Please return to page 23 and select a better answer.

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This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

YOUR ANSWER --- A

Incorrect. You're not following instructions.

The area of a circle is directly proportional to the square of the radius, or $A = kr^2$. You tripled r but then forgot to square it.

The expression should read: $A = k(3r)^2$. What do you get when you square 3r? You don't get $3r^2$ as this answer says.

Please return to page 53 and select another answer.

You have now completed the study portion of Lesson 6 and your Study Guide Computer Card and AV Computer Card should be properly punched in accordance with your performance in this Lesson.

You should now proceed to complete your homework reading and problem assignment. The problem solutions must be clearly written out on 8½" x 11" ruled, white paper, and then submitted with your name, date, and identification number. Your instructor will grade your problem work in terms of an objective preselected scale on a Problem Evaluation Computer Card and add this result to your computer profile.

You are eligible for the Post Test for this Lesson only after your homework problem solutions have been submitted. You may then request the Post Test which is to be answered on a Post Test Computer Card.

Upon completion of the Post Test, you may prepare for the next Lesson by requesting the appropriate

1. study guide
2. program Control Matrix
3. set of computer cards for the lesson
4. audio tape

If films or other visual aids are needed for this lesson, you will be so informed when you reach the point where they are required. Requisition these aids as you reach them.

Good Luck!

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YOUR ANSWER ---- B

Not quite. It might be well for you to draw a picture of the situation, second by second.

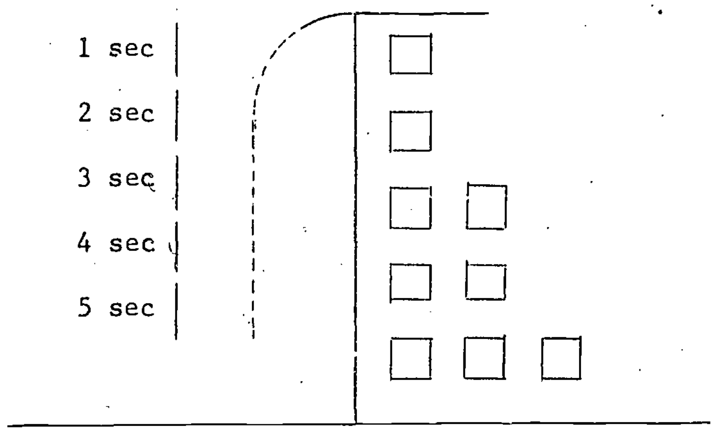

Figure 6

If you find the distance fallen in 4 sec and subtract this from the height of the building, what are you really determining?

(32)

A The distance of the stone from the top of the building after 4 sec.

B The distance of the stone from the ground after 4 sec.

YOUR ANSWER --- B

You are correct. The body whose graph of motion is shown in this drawing starts from rest because the graph goes through the origin of the axes; its acceleration is uniform because the graph is a straight line showing equal speed increases in equal times.

Referring to notebook entry 1(e) for this lesson, you see that the acceleration of the body in Graph 2 of Figure 4 on page 18 is given by the slope of the graph. The slope may be written as $\Delta v/\Delta t$, or:

$$a = \Delta v/\Delta t$$

For a body starting from rest, we can simplify this somewhat. Refer to Figure 5.

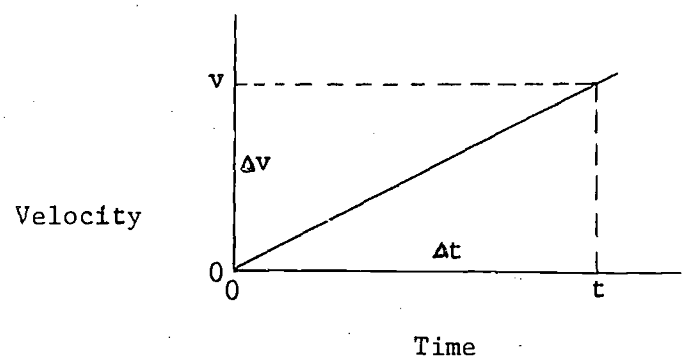

Figure 5

The dotted lines have been drawn in to show the <u>final velocity</u> (v) that has been reached after a certain <u>time</u> (t) has passed. The intervals Δv and Δt are also conventionally labeled. You will remember that Δv is the difference between the velocity at the end of a trip and the velocity at the start of the trip. ($\Delta v = v_2 - v_1$.) But since this body starts with an initial velocity of zero, then $v_1 = 0$ and v_2 = the final velocity which we symbolize by <u>just plain v</u>. Thus, for this case we can say that Δv, the change of velocity, is $v - 0$. Now when zero is subtracted from <u>any</u> number, what is the result?

(13)

A The result is zero.

B The number is unchanged.

YOUR ANSWER --- B

This statement is not incorrect. The relationship among t, v, and a is being questioned in this statement. So we write an equation involving these three terms. The equation obviously is:

$$a = v/t$$

If time is constant, then we have:

$$a = v/k$$

We want the k alone on one side of the expression, so we cross both the k and the a in the usual manner to obtain:

$$k = v/a$$

Thus, we see that there is a constant ratio, signifying a direct proportion; this means that v is directly proportional to a, or in words, that the final velocity is directly proportional to the acceleration if various bodies moving with various accelerations are allowed to travel for the same length of time.

Please return to page 78 and try to find the incorrect statement.

YOUR ANSWER --- A

Your error lies in failure to handle the units properly. If you take the time to substitute units as well as numbers in the equation, the chances of this type of error will be greatly reduced.

$$a = \frac{v}{t} = \frac{24\ \text{ft/sec}}{3\ \text{sec}}$$

If you find difficulty in handling units in fractional form as above, then simplify it by the usual process of converting the division to multiplication by the reciprocal of the denominator. Thus:

$$a = 24\frac{\text{ft}}{\text{sec}} \times \frac{1}{3\ \text{sec}}$$

You should see at once that the denominator contains two time units. In the answer you chose, the denominator contains only one; hence it cannot be correct.

Please return to page 110 and select the answer that fits this situation best.

YOUR ANSWER --- A

You are correct. Whenever two variables like d and t^2 are related to ~~ch other by a constant like k, you may be sure that a proportionality exis..

The question is, what kind of proportion, direct or inverse, does the relation $d = kt^2$ imply?

Well, let's see. If t^2 is increased, what happens to d? If the equation is solved for k, we then obtain $k = d/t^2$. This shows that there is a constant ratio for values of d and t^2. But if corresponding values form a constant ratio, then what kind of proportion exists between d and t^2?

(27)

A The implied proportion is inverse.

B The implied proportion is direct.

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YOUR ANSWER --- C

You handled most of the problem correctly but did not complete the final step. You correctly determined the distance that the stone fell at the end of 5 sec, but you did not find out how far from the ground the stone would be at this instant.

Please return to page 149 and, after completing the necessary step, select the correct answer.

YOUR ANSWER --- C

You are correct. For the initial circle, $A = kr^2$, when the radius is tripled, the expression is, $A = k(3r)^2$ or $A = 9kr^2$, and since the second expression is 9 times as large as the initial expression, the area must go up by a factor of 9.

Before continuing, please turn to page 176 in the blue appendix.

You'll be getting lots of additional practice in handling direct proportions as we go more depply into physics. You will also encounter <u>inverse proportions</u> very often. If you understand the foregoing material on direct proportions, the inverse proportion should not cause any trouble whatever.

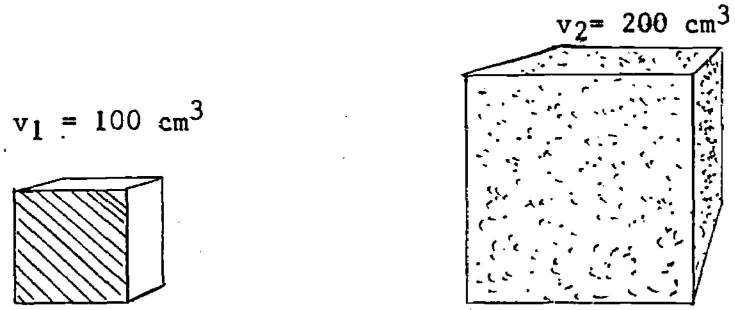
To help you recall the significance of an inverse proportion, we'll introduce a concept in physics that makes use of it.

Please turn to page 105 to continue.

The density of a substance is defined as its mass per unit volume. Cork is less dense than lead because a 1 kg mass of cork has a much larger volume than a 1 kg mass of lead. In symbols we can write:

$$D = \frac{M}{V}$$

where D = density, M = mass, and V = volume. For convenience, we'll measure the mass in grams (g), the volume in cubic centimeters (cm^3), and the density in g/cm^3 or grams per cubic centimeter.



Solid Rubber Mass
96 g

Foam Rubber Mass
96 g

Figure 3

Referring to Figure 3, we see at the left a mass of solid rubber of 96 g with a volume of 100 cm^3. Using the equation, what is the density (D) of the solid rubber?

Write the solution; then turn to page 168.

YOUR ANSWER --- B

This isn't a good way to do it.

If we committed an error in the original derivation, chances are that we would make the same mistake as we worked the derivation backwards. Furthermore, this is a difficult process in many cases and consumes much more time than it's worth.

There is a much better way to do this. Try to remember what it is.

Please return to page 10 and find the right answer.

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YOUR ANSWER --- B

No, that's not right.

It is common symbol notation to omit the "times" sign and merely place one symbol after the other. For example:

a x b may be written as ab.
3 x 5 may be written as (3)(5).
$m^2 \times n^3$ may be written as m^2n^3.

When two quantities are to be <u>divided</u>, however, we always use either a fraction bar or a diagonal slash bar. To show I_1 divided by R_1, we could write it in either of these two ways:

$\frac{I_1}{R_1}$ (fraction bar)

I_1/R_1 (diagonal slash bar)

Thus, I_1R_1 does not signify division, nor does I_2R_2.

Please return to page 89 and select the correct answer.

CORRECT ANSWER: The time for the 12-inch trip would be closest to 2 seconds.

When this experiment is performed with very precise timing equipment, it is found that the 12-inch trip requires almost exactly 2 seconds if the 3-inch trip requires 1 second. (There is another factor present which does not permit the 12-inch trip time to be exactly 2 seconds. Discussion of this factor is beyond the scope of this course.)

Tabulating the data we get:

Trip length (d)	Time (t)
3 inches	1 second
12 inches	2 seconds

Now the question is, "Were these results predictable from the proportion $d = kt^2$?"

First, the body started from rest in each trial. Right?

Second, the acceleration produced by gravitational force acting on the marble may be considered quite uniform for this experiment.

Therefore, the distance traveled should be directly proportional to the square of the time. Is it?

(25)

A Yes.

B No.

C I don't know.

YOUR ANSWER --- A

You are correct. For this special case, Δv reduces to v, the final velocity; and Δt reduces to t, the time required for the entire trip.

Since this is the first special equation of motion for the case of a body starting from rest and accelerating uniformly, we'll make it the next notebook entry.

NOTEBOOK ENTRY
Lesson 6

3. Equations for a body starting from rest and accelerating uniformly.

(a) The acceleration is the ratio of the final velocity to the trip time or:

$$a = v/t$$

Until you use an equation to solve a few practical problems, it can have little meaning for you. So let's do a few of these.

A ball started from rest, accelerated uniformly down an inclined plane, and attained a velocity of 24 ft/sec in 3 sec of travel time. What was its acceleration?

(15)

A The ball's acceleration was 8 ft/sec.

B The ball's acceleration was 8 ft/sec^2.

C The ball's acceleration was 8 ft.

YOUR ANSWER --- A

Your answer indicates that you are handling the proportion as though the distance were proportional to the time of travel. That is, you are apparently saying that, since 8 sec is an interval that is 4 times as long as 2 sec, then the car must travel 4 times as far during the longer interval (28 m x 4 = 112 m). This reasoning would be all right _if_ d = kt. But it doesn't! _We know that $d = kt^2$_. Remember, this car is _accelerating_ all the time.

Please return to page 138 and work out the proportion correctly.

YOUR ANSWER --- A

That's right; the unit check is successful. Here it is:

$$\frac{m^2}{sec^2} = \frac{m}{sec^2} \times m$$

$$\text{So} \quad \frac{m^2}{sec^2} = \frac{m^2}{sec^2}$$

NOTEBOOK ENTRY

Lesson 6

(Item 3)

(c) When a body starts from rest and accelerates uniformly, the distance it covers may also be found from the relation:

$$d = \frac{v^2}{2a}$$

If this equation is solved for v, it is extremely useful for determining the final velocity of a body dropped from a given height near the Earth's surface.

$$v = \sqrt{2ad}$$

Some time back we solved a problem in which a ball was projected vertically upward with an initial velocity of 96.0 ft/sec. We did this in two steps, remember? We found the time required for the ball to reach maximum height; then we determined the actual height to which it had risen at the top of its ascent. The maximum height turned out to be 144 ft. Now we can do the same problem using $d = v^2/2a$ in a single step instead of in two separate ones.

Run through this problem now using our new equation. The ball is treated as a falling object. If you do it right, your answer will be 144 ft.

Please turn to page 60 and check our solution against yours.

YOUR ANSWER --- B

No, that's not right. Group 2 consists of these expressions:

$$vt = k \qquad\qquad P = k/V$$

We'll let you decide for yourself what type of proportion is implied in $vt = k$.

In the case of $P = k/V$, you'd be able to recognize it more easily if you transposed the V to the left side. Most people do this by multiplying both sides of the equation by V. When you do this, you get $PV = k$. You should be able to recognize what type of proportion this is.

Please return to page 146 and select another answer.

YOUR ANSWER --- A

You are correct. The "sec^2" in the numerator cancel out, leaving $m = m/2$, but since 2 is a pure number without units, the unit check resolves itself into $m = m$. Thus, we are satisfied that the equation is correct dimensionally.

Students who reach this point in their work sometimes begin to lose sight of the important objective of deriving equations. Let's review this. For the physicist, an equation of symbols is merely a shorthand way of describing some phenomenon, principle, or event in the real world. Unlike the mathematician, the physicist does not view symbols as ends in themselves; each symbol has a physical meaning, and the equation is useless if it cannot be translated into things or happenings in the world in which we live.

Suppose you knew that your automobile was rated by the manufacturer as being capable of accelerating at the rate of 10 ft/sec^2 from a stop. Suppose further that you were curious to know how far you could go (from a stop) if you allowed the car to accelerate for 4 seconds. We have derived and checked the relation $d = \frac{1}{2}at^2$. We also have the equations $a = v/t$, $\overline{v} = d/t$, and $\overline{v} = v/2$ for motion starting from rest and accelerating uniformly. Which one of the following equations contains all the terms needed to solve the car problem?

(21)

A $d = \frac{1}{2}at^2$

B $a = v/t$

C $\overline{v} = v/2$

D $\overline{v} = d/t$

YOUR ANSWER --- C

You are correct. If d is multiplied by 6, then v will be multiplied by $\sqrt{6}$. Since the square root of 6 = 2.45, then v will increase by this factor.

Many students begin to develop the feeling at about this point that physics is nothing more than "disguised" mathematics. Actually, this is not at all true. It happens that fundamental concepts and understandings are based upon descriptive or verbal logic. Nevertheless, we like to express these quantitatively as possible in mathematical terms because the process is less time-consuming and certainly more exact and rigorous. A certain word may have more than one meaning, each of which may be vague or ambiguous; a mathematical symbol, once defined, can mean only one thing. You should always view mathematics as the language of science, not the science itself.

Please turn to page 180 in the blue appendix.

YOUR ANSWER --- D

You jumped to conclusions without giving enough attention to the facts depicted by Graph 4 of Figure 4 on page 18.

This is a distance-time curve. It is a straight-line graph, so it shows that the body is covering equal distances in equal times. But what must be true of the speed of a body that covers equal distances in equal times? It must be moving with constant speed. But if the body moves with constant speed, what is its acceleration? Zero!

This means that a body that accelerates at all could not give a straight-line d-t curve. It's really important to keep clear in your mind the difference between a v-t curve and a d-t curve.

Please return to page 18. Select the correct graph this time.

YOUR ANSWER --- C

You are correct. Simple cross-multiplications and transposition of terms gives us this expression by means of which we can now obtain the time required for the flour sack to fall to Earth.

The altitude of the balloon is 7.84 km; $a = g = 9.8$ m/sec^2.

To keep our units consistent, what change must be made in one of these quantities? Well, we could change the units of g to km/sec^2 if we wished but a better idea would be to change 7.84 km to meters.

Now work out the problem and obtain the answer. How long does it take the flour sack to fall to Earth?

Keep your answer handy as you turn to page 22 to check your result against ours.

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This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

YOUR ANSWER --- B

You are correct. The relationships given in these items are:

(b) $\bar{v} = d/t$ or average speed equals distance over time. This is applicable to any type of motion, including that of a body starting from rest and accelerating uniformly.

(c) $\bar{v} = v/2$ or the average speed of a body that starts from rest and accelerates uniformly is equal to its final velocity divided by 2.

Since both of these equations are valid for a body starting from rest with uniform acceleration, then the $\bar{v}$ factor must mean exactly the same thing in both relationships. These equivalent equations will help us derive a new and very useful expression which relates acceleration, time, and <u>the distance covered</u>.

Things equal to the same thing are equal to each other. Using this axiom and the equations, select the only valid expression from those below:

(18)

A $d = 2vt$

B $v = dt/2$

C $2d = v/t$

D $d = vt/2$

YOUR ANSWER --- A

The kind of expression shown above may be new to you, but you can interpret it properly if you follow through a few logical steps.

Consider the first two terms only: $I_1R_1 = I_2R_2$. What does this mean?

It states simply that the product I_1R_1 is equal to the <u>same number</u> as the product I_2R_2. Saying this another way, as the quantity R (whatever it may be) changes from a value R_1 to a value R_2, then the quantity I (whatever <u>it</u> may be) changes from I_1 to I_2 <u>in such a way as to keep the product the same</u>. That is, as variations in R cause corresponding variations in I, the product of the corresponding pairs always gives the same answer. In other words, <u>the product of I and R is a constant</u>.

Refresh your memory by looking at your notes. Does a constant product indicate a proportionality at all? If it does, what kind of proportion is indicated?

Now please return to page 71 and see if the answer you want is among those given.

YOUR ANSWER --- B

You are correct. To find the factor by which v increases, you must take the square root of the factor by which d increases. So, if d is multiplied by 25, then v will be multiplied by $\sqrt{25} = 5$.

Suppose you solidify this concept by answering a question about case D of Figure 8 on page 144. The distance through which the ball falls in this case is 6 units. In short, the distance has now been increased by a factor of 6 as compared with that of case A.

How many times as large as it was initially (in case A) will v now be?

(44)

A 36 times as great.

B 6 times as great.

C 2.45 times as great.

YOUR ANSWER --- D

Both answers are not correct.

We shall not divulge your error to you. We want you to go back and rework the problem more carefully, being sure you read exactly what is given and what is to be determined.

Please return to page 39 and follow the instructions. Then select a better answer.

CORRECT SOLUTION:

(1) Since $a = v/t$, then $t = v/a$.

(2) Substituting: $t = \frac{78.4 \text{ m/sec}}{9.80 \text{ m/sec}^2}$

(3) 78.4 divided by 9.80 equals 8.00. This is the numerical portion of the answer. Next, the units

$\frac{\text{m/sec}}{\text{m/sec}^2}$ should be written in reciprocal form like this:

$$\frac{\cancel{\text{m}}}{\cancel{\text{sec}}} \times \frac{\text{sec}^{\cancel{2}}}{\cancel{\text{m}}} = \text{sec}$$

Thus, the final answer is 8.00 sec.

The acceleration of a body in free-fall--9.80 m/sec^2--is an experimentally determined value. It varies from place to place on the Earth's surface, but we take 9.80 as a nice "round" number that is quite close to the actual value in the temperate latitudes. It is a handy constant to remember because it crops up in many places.

It will also be helpful to you to know the value of this gravitational acceleration (often symbolized as g rather than a for this special case) in ft/sec^2. Since there are 3.28 ft in one meter, then gravitational acceleration, g, in English units is obtained from: $g = 9.80 \text{ m/sec}^2 \times 3.28 \text{ ft/m} = 32.2$ ft/sec^2. For problem work, we can round this off to 32 ft/sec^2.

With what speed will a ball strike the ground if it starts from rest and falls freely for 4 sec? Write out the solution in ft/sec and then check by turning to page 81.

YOUR ANSWER --- C

You forgot an important step.

The required equation is, of course, $d = \frac{1}{2}at^2$.

You substituted correctly but forgot to divide the numerator by 2.

Do the problem over. Then return to page 34 and select a better answer.

YOUR ANSWER --- A

You are correct. Here's the solution:

Given	Find	Equation
Bldg. ht = 550 ft $a = g = 32\ ft/sec^2$ $t = 5$ sec	The distance of the stone from the ground after 5 sec of fall.	$d = \frac{1}{2}at^2$

Substitutions

$$d = \frac{32\ ft/sec^2 \times 25\ sec^2}{2}$$

$$d = 16\ ft/\cancel{sec^2} \times 25\ \cancel{sec^2}$$

$$d = 400\ ft$$

The stone fell 400 ft in 5 sec. The building is 550 ft high, so the distance of stone from ground is 550 ft - 400 ft = 150 ft.

We sometimes wish to determine how far something falls during one particular second. For example, how can we find out how far the stone in our problem fell during the fifth second?

(31)

A We already know the distance fallen in 5 sec. Then find the distance fallen in 6 sec and subtract one from the other.

B Find the distance fallen in 4 sec and subtract this from the height of the building.

C We already know the distance fallen in 5 sec. Then find the distance fallen in 4 sec and subtract one from the other.

YOUR ANSWER --- C

You are correct. That's the way to do it.

We can write the equation we just derived ($d = v^2/2a$) in two alternative forms by transposing terms in the approved manner. Thus:

$$d = \frac{v^2}{2a} \text{ is equivalent to } v^2 = 2ad \text{ is equivalent to } v = \sqrt{2ad}$$

It is usually easier to do a unit check on a product type of equation, so let's perform the operation on $v^2 = 2ad$. Remember how this is done? Use the mks units: m/sec for v, m/sec^2 for a, and m for d.

Do the check. When the proper units are substituted, do you wind up with a dimensionally valid equation?

(40)

A Yes.

B No.

C I don't remember how to perform a unit check.

YOUR ANSWER --- A

No, it doesn't. Note that when the timing starts (refer to Figure 4 on page 18), that is when t = 0, the body already has a definite speed. This is shown by the fact that the curve intercepts the Y axis at some value greater than zero. This means, of course, that the body does not start from rest.

You can select the correct curve by thinking out the answer to the question, "Where must the curve start to indicate an initial rest condition?"

Please return to page 18 and look over the other curves carefully before making another choice.

YOUR ANSWER --- C

No, that answer is not right.

Let's see why it isn't. Since things equal to the same thing are equal to each other, then

$$\frac{d}{t} = \frac{v}{2}$$

because each side of this equation is equal to $\overline{v}$.

So far, so good. In the expression you chose as valid, the equality above is presumably solved for 2d.

In order to get 2d alone on the left, it would be simplest to cross-multiply in the usual manner. But if you do this, you obtain:

$$2d = vt \quad \text{(not } 2d = v/t\text{)}$$

So the result given above is definitely not a valid one.

Please return to page 120 and select another answer.

YOUR ANSWER --- B

There's an error in your algebra. Let's find it.

Starting with $d = \frac{1}{2}at^2$, if we want to solve for t, we shall want this factor alone on one side of the equation; it doesn't matter which side. So, we'll leave the t^2 where it is and start by cross-multiplication. Thus, the 2 (a divisor in the first place) may be crossed to the other side and become a multiplier of the d. This gives us:

$$2d = at^2$$

We want the t^2 all alone; hence we will divide both sides of the equation by a, thus obtaining:

$$\frac{2d}{a} = t^2$$

Interchanging right- and left-hand terms, we can write:

$$t^2 = \frac{2d}{a}$$

The equation is thus solved for t^2, not for t. Now, how can we reduce the expression to a solution for t?

You should now be able to choose the correct answer. Please return to page 161 and make your selection.

This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

YOUR ANSWER --- B

You are correct. As long as you don't change the width, then the ratio of area to length must always come out to be the same number, in this case, 2.0 cm.

NOTEBOOK ENTRY

Lesson 6

2. Proportions

(a) Two quantities are in direct proportion if multiplying (or dividing) one of them by a given factor causes the other to be multiplied (or divided) by the same factor.

(b) Two quantities are in direct proportion if their ratio is a constant.

Let's express this algebraically. When you see this statement: $y/x = k$ in which k = a constant, then you should immediately recognize that y is directly proportional to x. From now on we shall refer to k as a proportionality constant.

Even more often, you will see this little equation written in a slightly different form like this: $y = kx$

Make it a point to read out this expression as "y is directly proportional to x", not as "y equals k times x."

All right. If $B = kA$ and if the value of A is reduced to 1/3 of its initial size, then which one of the following is true?

(3)

A The new B will be 3 times as large as the old B.

B The new B will be equal to the old B, since k will change to a new value.

C The new B will be 1/3 as large as the old B.

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YOUR ANSWER --- B

You're half right and half wrong! You recognized that there was an inverse proportion here, but you failed to take into account the fact that the distance (d) is squared.

In stating such a proportionality in words, all mathematical information must be presented so that the reader has no misunderstanding of the relationship. That square is <u>very</u> important.

Please return to page 37 and select a more consistent answer.

YOUR ANSWER --- D

Somewhere in working through the algebra on this, you inverted your terms.

Starting with $d = \frac{1}{2}at^2$, if we want to solve for t, we shall want this factor all alone on one side of the equation; it doesn't matter which side. So, we'll leave the t^2 alone and start with cross-multiplication. Thus, the 2 (a divisor in the first place) may be crossed to the other side and become a multiplier of the d. This gives us:

$$2d = at^2$$

To obtain the t^2 alone on the right side, we can divide both sides by a. Thus:

$$\frac{2d}{a} = t^2$$

You see, then, that the d is in the numerator and the a in the denominator. That's why we said initially that you had inverted the terms.

Please return to page 161 and complete this solution.

YOUR ANSWER --- A

You are correct. For the first set of tabulated data, d/t^2 gives $3/1 = 3$; for the second set, we get $12/4 = 3$. Since the same number is obtained, then d and t^2 are in constant ratio and hence are directly proportional.

The usefulness of this concept of proportionality may be nicely shown by the following practical situation: Let us assume that a special racing car can be accelerated uniformly by a powerful engine. Starting from rest, the car travels a distance of 28 meters after 2 seconds. Assuming no change in the acceleration, how far will it have gone at the end of 8 seconds of travel?

Note that the acceleration has not been stated explicitly. Although you have enough facts to find the acceleration if you so desire it, the problem can be solved by the use of proportions, without determining the acceleration at all.

Work out the problem; then select the one answer below that corresponds with yours.

(28)

A 112 meters

B 224 meters

C 448 meters

CORRECT ANSWER: Certainly, the time of rise and the time of fall will be exactly the same. Look here: if $d_{rise} = d_{fall}$ (and this is true) and if $a_{rise} = a_{fall}$ numerically (and this is also true), we can write:

Using the equation $t = \sqrt{\frac{2d}{a}}$

Time of rise $= t_{rise} = \sqrt{\frac{2d_{rise}}{a_{rise}}}$

Time of fall $= t_{fall} = \sqrt{\frac{2d_{fall}}{a_{fall}}}$

Since $d_{rise} = d_{fall}$ and $a_{rise} = a_{fall}$, then $t_{rise} = t_{fall}$.

Thus, you see, we can find the maximum height of the ball by considering the fall path, with the ball starting from rest and accelerating uniformly at 32.0 ft/sec^2.

Before continuing, please turn to page 179 in the blue appendix.

O.K. The time of fall is 3.00 sec. Now determine the maximum height reached by the ball.

The maximum height reached by the ball was which of these?

(36)

A 48.0 ft

B 144 ft

C 288 ft

YOUR ANSWER --- A

You probably didn't read this one carefully. Adding <u>or</u> subtracting zero f a number does not give a sum or difference of zero. How much is 15 plus . How much is 27 minus 0?

Actually, the only operation in arithmetic using a zero as the operator that does yield zero as an answer is multiplication. Thus, $54 \times 0 = 0$. But this is not true of addition and subtraction, as you know.

So, please return to page 98 and choose the other answer.

YOUR ANSWER --- D

You are correct. Good work. Since things equal to the same thing are equal to each other, we have:

$\overline{v} = d/t$ and $\overline{v} = v/2$ so that we can write: $\frac{d}{t} = \frac{v}{2}$.

Next, we get d alone on the left side by transposing the t (multiplying through by t) and are then left with $d = vt/2$.

Remember that we are trying to derive an expression which relates acceleration (a), time (t), and distance (d). As you look at

$$d = vt/2$$

you see that v rather than a is present. So, we should like to replace v with a in a legitimate mathematical way. When you reach a point like this in a derivation, you must try to think of another equation which relates v and a. Then, you can solve this other equation for v and replace it with its equality. We have just such an expression in that for a body starting from rest and accelerating uniformly:

$$a = v/t$$

Solving this for v, we obtain $v = at$. So, since v and at are identities, we can replace the v with at in $d = vt/2$.

Suppose you do that on scrap paper. Which one of the following expressions is obtained by this manipulation?

(19)

A $d = at/2$

B $d = at^2/2$

YOUR ANSWER --- B

Density is mass per unit volume, or if you prefer, mass divided by volume. Even though the amount of rubber or mass of the block remains the same in the "foaming" process, the volume does change. This means that the new fraction M/V must be different after V has been altered.

Since density depends upon two quantities, the density of a given body must change if either the mass or the volume changes.

Please return to page 168 and select another answer.

YOUR ANSWER --- C

Your choice is correct. There is an error in the statement. Let's find it. We are questioning the relationship among a, v, and d. To establish the truth or falsity of the statement "If acceleration is constant, then the final velocity of a body is directly proportional to the distance the body travels," we want to write an equation which includes all three of these terms. The only one that fits the requirement is:

$$v = \sqrt{2ad}$$

Since the acceleration is constant, we may replace 2a by the sumbol k:

$$v = \sqrt{kd}$$

For convenience, we may bring the constant outside the radical because the square root of a constant is still constant:

$$v = k\sqrt{d}$$

Leaving the k on the right side, we then transpose the $\sqrt{d}$ to the left, bringing it down into the denominator:

$$\frac{v}{\sqrt{d}} = k$$

Here we have a constant ratio, signifying a direct proportion. That is, v is proportional to the square root of d. In words, the velocity acquired by a body in uniform acceleration is directly proportional to the square root of the distance that it travels.

Please go on to page 144.

This is the first time a square root has entered into a proportionality in our work, so we should investigate it briefly.

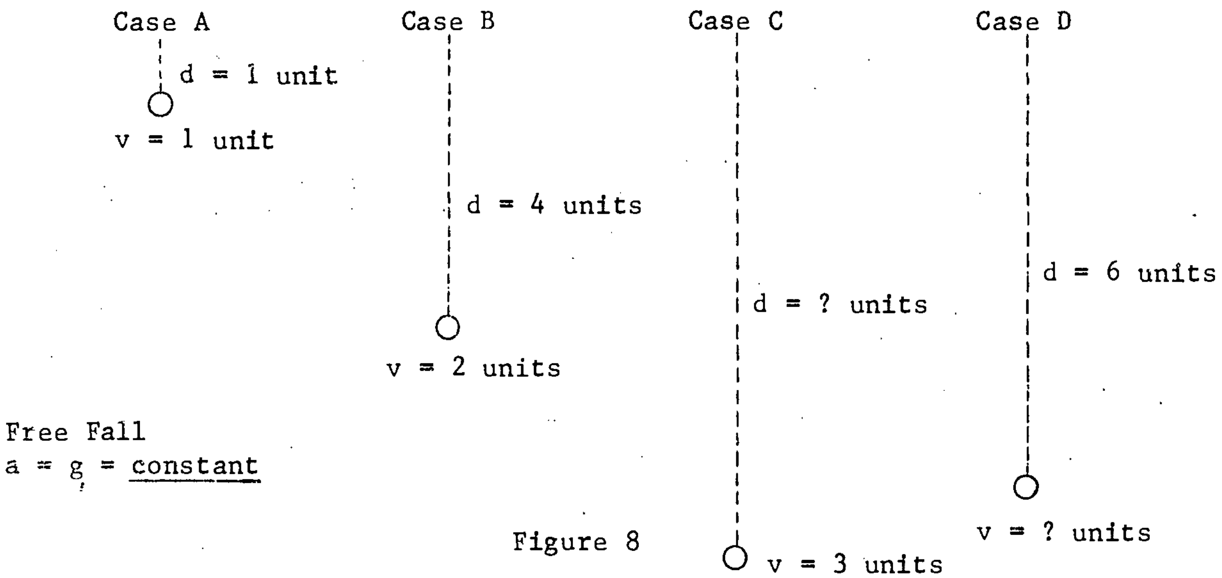

Figure 8

Refer to Figure 8. Using arbitrary units for <u>d</u> and <u>v</u>, we set up case A so that the body falls 1 unit of distance and, in the process, acquires 1 unit of velocity. From $v = \sqrt{2ad}$ and the fact that $2a$ is a constant, for case A we see that $1 = \sqrt{k \times 1}$ so that the value of $k = 1$.

This simplifies our arithmetic substantially. Thus, case A may be set forth as: $1 = \sqrt{1 \times 1}$, where d = 1 unit, v = 1 unit, and k = 1.

Next consider case B. Ask yourself, "What value must <u>d</u> have in order to allow the ball to acquire a final velocity of <u>2 units</u>?" Well, if d = 4 units, then <u>v</u> will be 2 units because:

$$2 = \sqrt{1 \times 4}$$

Now refer to case C. If we want <u>v</u> to be 3 units, what value must <u>d</u> have? In other words, for constant acceleration, from what height must the ball be released so that it acquires a velocity of 3 units? Using the same technique, we could write:

$$3 = \sqrt{1 \times ?}$$

What number would you replace the ? with? Keep the answer in mind as you turn to page 173.

YOUR ANSWER --- A

Your algebra is fairly good. The trouble is that you forgot a step near the end and omitted an important consideration. We'll accept the right side of the equation (i.e. 2d/a), but we insist that this is not equal to just plain t. Work it over again. What did you forget to do?

Please return to page 161. Choose one of the other answers, the one that shows your omission properly corrected.

YOUR ANSWER --- B

You are correct. Here's the reasoning: if the products--I_1R_1, I_2R_2, I_3R_3--are equal to each other, then the product of I and R (IR) for any values of these quantities (whatever they may symbolize) must be constant. But, if the product of two quantities is constant, then these quantities are inversely proportional. (Notes: 2(d).)

The next question is a check on your understanding of how to recognize direct and inverse proportions from the position of the proportionality constant.

Three groups of relationships are given below. You have already discovered that you don't have to know the meanings of the symbols for the variables; all you must know is that

$$k = a \text{ constant.}$$

With this knowledge you should be able to recognize direct and inverse proportions. Take your time. Study the groups; then select the correct statement from those given below.

Group 1	Group 2	Group 3
$f/m = k$	$vt = k$	$W = kf$
$pv = k$	$P = k/V$	$E/w = k$

(10)

A Group 1 contains two inverse proportions.

B Group 2 contains a direct and an inverse proportion.

C Group 3 contains a direct and an inverse proportion.

D Group 2 contains two inverse proportions.

YOUR ANSWER --- A

You forgot a very important step.

The equation you want, of course, is $d = \frac{1}{2}at^2$.

You substituted correctly but forgot to square the value for time.

Run through it again; then return to page 34 and select the correct answer.

YOUR ANSWER --- C

You have probably forgotten how to recognize a direct or inverse proportion. Let's run through the procedure quickly to re-establish the method in your mind.

In this particular case, we are basing our thinking on the equation $d = \frac{1}{2}at^2$. We started the body from rest, thus fulfilling one of the two conditions for which this equation is valid. We have been told that acceleration produced by gravitational force is uniform; this meets the second condition.

If acceleration is uniform, then a must be a constant. The divisor, 2, is also a constant; hence a/2 is a constant. This permits us to write the distance equation as $d = kt^2$ where k has replaced a/2. All this is perfectly legitimate, mathematically and logically. But the moment that you see a relation in the form $d = kt^2$, what should this suggest to you?

(26)

A It should suggest that d and t^2 are somehow proportional.

B It should suggest that d and t^2 are related but not necessarily proportional in any way.

YOUR ANSWER --- B

You are correct. Since distance is proportional to the square of the time, then the new distance will be equal to $(9)^2d$ or 81d.

To give you a better "feeling" for the significance of the relation between distance and time in uniformly accelerated motion starting from rest ($d = \frac{1}{2}at^2$), you will want to do a few typical problems. Try this one first. (Please use the suggested format: list what is given in terms of their symbols, state what you are to determine, write the necessary equation, and, if necessary, solve it in its literal form for the unknown before substituting.)

A building is 550 ft tall. A stone dropped from the top of the building is allowed to fall for 5 sec. How far from the ground is the stone at the end of the 5-sec interval? (The acceleration due to the gravitational force may be considered uniform. For the actual values of g, refer to notebook entries 3(a)(1) and 3(a)(2).)

Choose the correct answer from those given below.

(30)

A The stone will be 150 ft from the ground at the end of 5 sec.

B The stone will be 470 ft from the ground at the end of 5 sec.

C The stone will be 400 ft from the ground at the end of 5 sec.

YOUR ANSWER --- B

You are correct. In this case, we saw that $\Delta v = v - 0$. So, <u>for a body starting from rest and accelerating uniformly</u>:

$$\Delta v = v - 0 \text{ or } \Delta v = v.$$

Clearly, the same kind of thing occurs when we work with Δt. Since Δt is the difference between the final time and the time reading at the beginning of the trip, then: $\Delta t = t_2 - t_1$.

For this case, $t_1 = 0$ because the timing process starts with the clock hands reading zero time; therefore, t_2 is merely <u>t</u>, the time reading at the designated point. Thus:

$$\Delta t = t - 0 \text{ or } \Delta t = t.$$

We said we were going to write the definition of acceleration in a simpler form, applicable to a body that starts from rest and accelerates uniformly. Well, the definition of acceleration is:

$$a = \Delta v/\Delta t$$

So which of the expressions below is this simplest form?

(14)

A $a = v/t$

B $a = v/\Delta t$

C $a = \Delta v/t$

YOUR ANSWER --- A

You are allowing your attention to stray. You have not taken directions into account. When you reread the text material above the questions, you will see why the alternative choice is correct.

Please return to page 68.

YOUR ANSWER --- A

There is a serious error here.

Although you were perfectly right in substituting the <u>k</u> for the π, since π is a constant and may be considered to be the constant of proportionality, you altered the variable quantity r^2 in a way that will not give you the right answer.

In converting an equation into a proportionality, <u>you must never change or modify any of the variables</u>. In this example, the area is found by multiplying π by the <u>square</u> of the radius; this square term must be found in the proportionality statement without change.

Please return to page 77 and select another answer.

YOUR ANSWER --- C

Good! You are correct. As the distance doubles, the intensity is reduced by $\frac{1}{4}$; as the distance triples, the intensity is reduced by 1/9.

Before continuing, please turn to page 177 in the blue appendix.

To what fraction of its former value is the intensity reduced if the distance is increased 5 times?

Write your answer; then turn to page 18 to check it.

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YOUR ANSWER --- B

You are correct. Let us say that the answer to either of these two products is given by the letter k. We use k because this is the letter we have adopted to signify a constant or an unchanging number. Since the two products have the same answer, we can write:

$$I_1R_1 = k$$

$$\text{and} \quad I_2R_2 = k$$

You should remember that in our original expression we had: $I_1R_1 = I_2R_2 = I_3R_3 \ldots$, and so on. This states that whenever we change one of the quantities--for instance, R--then the other quantity, I, changes in such a way as to keep the product constant. As an example consider in the first instance that $I_1 = 8$, while $R_1 = 2$. For these values $I_1R_1 = 8 \times 2 = 16$. Now assume that we change the R value so that $R_2 = 4$. This will cause I_2 to decrease to 4 so that $I_2R_2 = 4 \times 4 = 16$. Now suppose we change the R value so that $R_3 = 16$. Since the product must still be 16, what value must I_3 assume?

Write your answer; then turn to page 27.

YOUR ANSWER --- B

This is not correct.

You must have forgotten that units can be manipulated just like numbers in unit calculations. Thus, in the unit equation:

$$m = \frac{\frac{m}{\cancel{sec^2}} \times \cancel{sec^2}}{2}$$

the "sec^2" factors can be canceled, leaving $m = \frac{m}{2}$.

Now, since 2 is a pure number, it is dimensionless, that is, it has no units. Hence, the 2 in the denominator cannot change the units in any manner and may be omitted from consideration. So we end up with m = m, and we have our dimensional check.

Please return to the original question by turning to page 175. Reread it once more; then select the alternative answer.

YOUR ANSWER --- B

You are correct. Both the change in velocity (Δv) and the acceleration (a) are <u>negative</u>.

Refer to the answer you just chose on page 68. In solving for Δt in this example, the (-) signs drop out, and the result is: $\Delta t = \underline{3}$ <u>sec</u>. This means that the ball will require 3 seconds to rise from the ground to its highest point if the initial velocity given to it by the thrower is 96.0 ft/sec.

Knowing the rise time, you can now determine the height to which it will rise before it comes to rest at the top of its flight. <u>But we have to be very careful here</u>. We want to use the equation $d = \frac{1}{2}at^2$ to find the height (d), but we know this equation applies <u>only to bodies that start at rest</u>. Well, the <u>body does</u> start from rest in its fall path, doesn't it? So, we'll make use of the fall path to find the maximum height of the ball above ground.

We can justify this quite rigorously. The ball is going to fall the same distance as it rose, so <u>d</u> is the same for both paths. It is going to pick up speed, or accelerate, at the same rate during its fall as it decelerated during its rise. In short, <u>a</u> has the same magnitude for both rise and fall, but it is <u>positive</u> for the fall path. So, if <u>d</u> and <u>a</u> both have the same magnitudes during rise and fall, what must be true of the time of rise as compared with the time of fall?

Keep your answer in mind as you turn to page 139.

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CORRECT ANSWER: The equation $d = \frac{1}{2}at^2$ should be solved for $\underline{t}$ as the unknown, since the problem requires that we find out "how long" a time is required for the sack to reach the ground.

Solve the equation above for $\underline{t}$ as the unknown. Which one of the answers below checks with your solution?

(33)

A $t = \frac{2d}{a}$

B $t = \sqrt{2ad}$

C $t = \sqrt{\frac{2d}{a}}$

D $t = \sqrt{\frac{2a}{d}}$

YOUR ANSWER --- A

This answer indicates that something is missing in your chain of reasoning.

Exactly what is meant by saying that one quantity is proportional to another? Don't we mean by this that as one quantity is made to vary or change the other quantity also varies in some systematic way? If a car travels with uniform speed, we say that the distance it travels is directly proportional to its speed. This statement also tells us that the car journeys farther if we allow it to travel for increasing amounts of time.

We're sure that you have no doubt at all that the intensity of illumination on, say, a movie screen decreases if the projector is moved farther and farther away. The question in your mind might be, "Does this decrease occur in a systematic, predictable manner?"

We have said that there is a mathematical relationship between illumination (I) and distance (d). This should suggest an answer to that question.

Sometimes a quantity may depend on another but may not be proportional to it. For instance, immediately after birth, a baby starts to gain weight. He may gain 4 ounces the first week, 6 ounces the second week, 5 ounces the third week, and 9.5 ounces the fourth week. Here, his weight depends on his age, but there is no mathematical equation to relate them; hence there is no proportionality.

Please go on to page 163.

Would you say that the size of a man's collar is directly proportional to his height?

Please turn to page 55 and see whether or not you agree with us on the answer.

YOUR ANSWER --- B

You are correct. When the general equation $a = \Delta v/\Delta t$ is solved for Δt, we have $\Delta t = v/a$. Since both Δv and a are negative (the car is losing speed),

$$\text{then } \Delta t = \frac{-88 \text{ ft/sec}}{-15 \text{ ft/sec}^2} = 5.9 \text{ sec},$$

There is an important underlying idea here. A change in velocity may be either positive or negative, depending on whether the body gains or loses speed; the same is true of acceleration. But changes in time (Δt) can never be negative because the passage of time is always from "now" to "later."

This matter of negative acceleration was introduced in the foregoing section to show what is meant by the (-) sign before either a or Δv and also to show that time, t, is never negative.

Now, returning to the relationship of time and velocity to distance as given by $d = \frac{1}{2}at^2$, we shall want to examine this expression somewhat more deeply. We know this equation applies only to cases where the body starts from rest and accelerates uniformly. If the acceleration is uniform, then it is constant; it does not vary from instant to instant; since 2 is also a constant number, then $a/2$, as a whole, must be constant. Then taking $a/2$ as a constant, we can write the equation above this way: $d = kt^2$ where k has replaced the constant quantity $a/2$. Now, what does $d = kt^2$ imply?

(24)

A It implies that distance is directly proportional to time.

B It implies that distance is directly proportional to the square of the time.

C It implies that distance is inversely proportional to the square of the time.

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YOUR ANSWER --- A

The distance is proportional to the <u>square</u> of the time, not to the time itself! You forgot to square the <u>9</u>.

Please return to page 38 and select the correct answer.

YOUR ANSWER --- B

Not true at all.

Why? Well, if things equal to the same thing are equal to each other, then

$$\frac{d}{t} = \frac{v}{2}$$

because the components of each side of this equation are equal to $\overline{v}$.

This much is surely correct. Now, in the expression you chose as valid, the equality above is presumably solved for v.

To get v alone on one side of the equation, you would have to eliminate the 2 on the right side. Most of us would probably do this by multiplying both sides by 2. Do this for yourself now. You will find that you do not get v = dt/2 as a result of this operation. Rather,

$$2 \times d/t = v/2 \times 2$$

This leaves:

$$2d/t = v$$

This result is certainly not the one expressed in this answer.

Please return to page 120 and select another answer.

CORRECT SOLUTION:

$$D = \frac{M}{V} \quad \text{so} \quad D = \frac{96\ g}{100\ cm^3}; \qquad \text{hence } D = 0.96\ g/cm^3$$

Thus, one cubic centimeter of rubber "weighs" 0.96 g.

Now suppose a rubber processor converts this piece of rubber to foam rubber without changing the quantity of material in it. In short, the mass will remain constant while its volume is increased by "foaming" it. Suppose further that the volume just doubles in the process. Thus, the original volume was $V_1 = 100\ cm^3$, while the new volume is $V_2 = 200\ cm^3$.

The volume has been doubled. What has happened to the density?

(6)

A The new density is 0.48 g/cm^3 which is exactly half the original density.

B The new density is exactly the same as before because the amount of rubber did not change.

C The new density is 1.92 g/cm^3 which is just double the original density.

YOUR ANSWER --- B

You are not applying the recognition rule for propotions.

We'll refresh your memory.

The easiest and best way to determine whether two quantities are directly proportional to each other is to find out whether or not their corresponding experimental values form a <u>constant ratio</u>.

We want to determine if distance is proportional to the square of time. Consider the first values: distance (d) = 3 in and time (t) = 1 sec. The test relation is:

$$\frac{d}{t^2} = ?\ \text{constant}$$

Substituting the first values for $\underline{d}$ and $\underline{t^2}$ we replace the $\underline{d}$ term with 3 (units may be omitted since they are consistent throughout the experiment) and the $\underline{t}$ term with 1 since $(1)^2 = 1$:

$$\frac{d}{t^2} = \frac{3}{1} = \underline{3}$$

Next, we replace the $\underline{d}$ and $\underline{t^2}$ with the second set of values, namely, 12 in and 4 sec^2 since $(2)^2 = 4$:

$$\frac{d}{t^2} = \frac{12}{4} = \underline{3}$$

Well? Is the distance traveled directly proportional to the square of the time?

Please return to page 109. You should certainly be able to select the correct answer now.

YOUR ANSWER --- D

No, there is nothing wrong with this statement. The relationship among t, d, and a is being questioned here. We therefore want an equation involving these three terms. Obviously we want:

$$d = \frac{1}{2}at^2$$

If the time is held constant, then $\frac{1}{2}t^2$ is a constant and may be replaced by k. Thus:

$$d = ka$$

To obtain the k alone on one side, we cross the a to the left and place it in the denominator. (This is the same as dividing through by a.)

$$d/a = k$$

We see that there is a constant ratio, signifying a direct proportion. Thus d is directly proportional to a or, in words, if the time of travel is held constant for a number of different bodies, then the distance covered by each one will be directly proportional to the acceleration of that body.

Return to the original set of statements by turning to page 78, please. Try again to find the incorrect one.

YOUR ANSWER --- A

Your algebra is faulty. Our basic equation is: $a = v/t$.

Before making the substitutions, solve the literal equation for the unknown as we have urged you to do. The unknown in this problem is v. Hence:

$$v = at$$

Now substitute the given numbers and units in this expression. Your answer will not be the one you first obtained.

Please return to page 16 and try again.

YOUR ANSWER --- B

Very good. You are quite correct. The solutions follow:

(a) To find total time of flight, find the time of rise and then double it. Thus:

$$t = \frac{v}{a} = \frac{-24.5\ \cancel{m/sec}}{-9.80\ \cancel{m}/sec^{\cancel{2}}} = 2.50\ sec$$

But total time = 2t; hence total time is 5.00 sec.

(b) To find maximum height of rise, find distance fallen from top of ascent in 2.50 sec as previously obtained.

$$d = \frac{at^2}{2} = \frac{9.80\ m/\cancel{sec^2} \times 6.25\ \cancel{sec^2}}{2} = 30.6\ m$$

Well, we're going into the last lap of this lesson now. We promised you that we would derive a single equation that would permit you to determine the height to which a ball rises, when you know the initial velocity and the value of the acceleration due to gravity. Let's review for a moment.

The expression $a = v/t$ relates acceleration, velocity, and time to each other for a body that starts from rest and accelerates uniformly. The expression $d = \frac{1}{2}at^2$ relates acceleration, distance, and time to each other for a body that starts from rest and accelerates uniformly. The expression we are about to derive, then, must therefore relate acceleration, __________, and ____________, if it is to enable us to determine the height of rise from a knowledge of initial velocity and acceleration. What are the missing words? Pick them out of the answers below:

(38)

A Distance and velocity.

B Distance and time.

C Velocity and time.

CORRECT ANSWER: The ? must be replaced by the number 9 in order to validate the equation.

Let's go over this verbally. If the velocity of a body in uniformly accelerated motion is to double, the distance it moves through must quadruple; if the velocity is to triple, the distance it moves through must increase 9 times; if the velocity is to quadruple, the distance must increase 16 times, etc.

Now look at it the other way. If the distance increases 4 times, v increases 2 times; if d increases 9 times, then v increases 3 times; if d increases 16 times, then v increases 4 times.

Thus, v is directly proportional to the square root of the distance through which the body moves.

All right. Now suppose we increase the distance to a value 25 times as great as before. By what factor will v be increased?

(43)

A v will be 25 times as great as it was initially.

B v will be 5 times as great as it was initially.

YOUR ANSWER --- A

No, not at all. The original conditions of the expression still prevail. If, to start with, we say that $I_1R_1 = I_2R_2$, we are telling you in no uncertain terms that the two products are equal to each other; hence both of them will have the same answer if the correct, applicable numerals are substituted for the symbols.

Please return to page 13 and select the correct answer.

YOUR ANSWER --- B

You are correct.

We have achieved our goal. We now have an expression that relates acceleration (a), time(t), and distance covered (d). The equation is frequently written in the form shown and also in the form: $d = \frac{1}{2}at^2$, which, of course, is identical in meaning. Before investigating the implications of this expression, we shall want to put it properly in our notes.

NOTEBOOK ENTRY

Lesson 6

(Item 3)

(b) When a body starts from rest and accelerates uniformly, the distance it covers is given by the relation:

$$d = \frac{at^2}{2} \quad \text{or} \quad d = \tfrac{1}{2}at^2$$

As a new equation, the first essential step for us is to test it dimensionally. This means that we must check to see that when actual quantities are substituted in it, the units will come out the same on both sides. IF THIS DOES NOT HAPPEN, THEN THE EQUATION IS WRONG.

We'll work with mks units. We'll use meters (m) for distance, meters per sec per sec (m/sec^2) for acceleration, and seconds (sec) for time.

Substituting units in $d = \frac{at^2}{2}$, we get: $m = \frac{\frac{m}{sec^2} \times sec^2}{2}$.

Do the units turn out to be the same on both sides?

(20)

A Yes.

B No.

WORKSHEET

Please listen to Tape Segment 1 of Lesson 6 before starting this Worksheet.

Always select answers for Worksheet questions by punching them out on the special AV Computer Card.

DATA ITEM A: Average diameter of Earth: 7,914 mi, or rounded back is 8,000 mi.
Average diameter of Sun: 864,400 mi, or rounded back is 800,000 mi.
Reduced ratio of diameters: 100 to 1.

DATA ITEM B: Volume of a sphere: $V = \frac{1}{6} \pi D^3$

DATA ITEM C: Volume-diameter proportion, sphere: $V = k D^3$

QUESTIONS

1. How many Earths could be fitted into one Sun?

   A 1 000
   B 10 000
   C 100 000
   D 1 000 000
   E 10 000 000

2. The volume of a sphere is directly proportional to the

   A cube of its surface area.
   B cube of its radius.
   C square of its surface area.
   D fourth power of its radius.
   E none of these is correct.

3. Regarding the shape of the Earth and the Sun,

   A both are perfectly spherical.
   B the Earth is an oblate spheroid but the Sun is a perfect sphere.
   C neither the Earth nor the Sun is a perfect sphere.
   D the Earth is a perfect sphere but the Sun is not a perfect sphere.
   E in both bodies, the equatorial diameter equals the polar diameter.

(next page, please)

4. A rubber ball with a volume of 512 cm^3 is compared with a steel ball-bearing with a volume of 1 cm^3. The ratio of diameters, rubber ball to steel ball, is

A 512 to 1.

B 22.3 to 1.

C 8 to 1.

D 4 to 1.

E $\sqrt{512}$ to 1.

5. Two perfect cubes are being compared. One of them has a length of side equal to 3.00 cm and the other has a length of side equal to 12.0 cm. The volume ratio, larger to smaller is

A 16 to 1.

B 4 to 1.

C $(16)^2$ to $(1)^2$

D $(8)^2$ to $(1)^2$

E $(4)^3$ to $(1)^3$

When finished, please return to page 104 of the STUDY GUIDE.

WORKSHEET

Please listen to tape Segment 2 of Lesson 6 before starting this Worksheet. Answer selections to be made on Computer Card.

DATA ITEM A: For electrically charged particles, the force of attraction is given by the relation:

$F = \frac{k}{r^2}$ in which F_e = force; k = proportionality constant; and r = separation distance of particles.

6. Two particles, separated by exactly 1 cm, each have a charge such that the force between them is 1 unit, -- a force of attraction. In order to reduce this force to 1/100 of its initial value, the separation would have to be

   A increased to 10 cm.
   B increased to 100 cm.
   C decreased to 0.10 cm.
   D decreased to 0.010 cm.
   E none of these is correct.

7. The gravitational force between two point masses can be shown to follow

   A the ratio of cubes.
   B the Law of Illumination.
   C the law of ratio of direct squares.
   D the law of inverse squares.
   E the law of magnetic poles.

8. The illumination falling on a screen varies inversely as the square of the distance from the light source provided that

   A the light source produces a parallel beam.
   B the intensity of the light source remains the same throughout the experiment.
   C the distance between source and screen remains constant.
   D the apparatus used to measure illumination contains scales which can be used to measure distances.
   E the light is monochromatic (single color).

When finished, please return to page 153 of the STUDY GUIDE and continue with the lesson.

WORKSHEET

Please listen to Tape Segment 3 before starting this Worksheet. As always, choose your answers by punching the AV Computer Card.

DATA ITEM A: The table below shows the values that g may assume at different locations on the surface of the Earth. All figures are given in MKS units, that is, in meters per second per second.

Alaska	9.82	Point Isable, Tex	9.79
Liberia	9.78	Greenland	9.83
Lisbon	9.80	New York City	9.80

9. Gravitational acceleration on the Moon would be roughly

A 1.63 ft/sec^2.
B 1.63 m/sec^2.
C 5.33 m/sec^2.
D 163 m/sec^2
E 9.80 m/sec^2

10. Near the surface of the planet Jupiter, the gravitational acceleration is roughly 26.5 m/sec^2. The time required for a body to fall 53.0 meters near the surface of Jupiter would be (Hint: Use $d = at^2/2$ in which g is substituted for a)

A 4 sec.
B $\sqrt{2}$ sec.
C 8 sec.
D 2 sec.
E $\sqrt{8}$ sec.

11. The gravitational acceleration in New York City may also be given as

A 32.0 m/sec^2
B 9.80 ft/sec^2
C 98 cm/sec^2
D 9800 cm/sec^2
E 980 cm/sec^2

When finished, please return to page 81 of the STUDY GUIDE for this lesson and continue.

WORKSHEET

Please listen to Tape Segment 4 of Lesson 6 before starting this Worksheet. Answer selections appear on the AV Computer Card.

12. For an object thrown vertically upward, the time required for it to ascend to the height it ultimately reaches is

A is shorter than the time required for it to fall back to its starting point from that height.

B is longer than the time required for it to fall back to its starting point from that height.

C is equal to the time required for it to fall back to its starting point from that height.

D is unrelated to the time required for it to fall back to its starting point from that height.

E is independent of the value of g at that point.

13. A stone thrown vertically upward from the surface of the Earth rises to its maximum height and then falls half way back to the surface. If this complete trip required 3 sec, what was the maximum height to which the stone rose?

A 96 m.
B 19.6 m.
C 14.7 m.
D 39.2 m.
E 44.1 m.

14. A ball thrown vertically upward from the surface of the Earth returns to its starting point in 10 sec. Its initial upward velocity must therefore have been

A 49.0 m/sec.
B 98.0 m/sec.
C 160 m/sec.
D 320 m/sec.
E 80.0 m/sec.

When finished, please return to page 139 of the STUDY GUIDE and continue with the lesson.

WORKSHEET

Please listen to Tape Segment 5 before starting this Worksheet. Use the AV Computer Card for indicating answers.

DATA ITEM A: Length of rifle barrel .... 2.00 ft.
Muzzle velocity .... 1,680 ft/sec.
Determine the acceleration, a.

15. Which one of the following equations should you use if you want to determine the time that the bullet spends in the barrel? (Note: You still don't know acceleration!)

A $\sqrt{\frac{2d}{a}} = t$ C $t = va$

B $d = 2at^2$ D $t = \frac{1}{2}ad^2$ E none of these.

16. What does the symbol "d" stand for in the equation you would use to find the time that the bullet spends in the barrel?

A The distance covered by the bullet after it leaves the barrel.
B The distance covered by the bullet before it enters the barrel.
C The distance the bullet falls toward Earth after it leaves the barrel.
D The distance the bullet rises toward its maximum height.
E The distance covered by the bullet in the barrel.

17. The time spent by the bullet in the barrel is

A 4.2 sec C 1/420 sec
B 1/42 sec D 1/840 sec
E none of these

18. Knowing the time spent by the bullet in the barrel and the final velocity, the acceleration is

A 706 ft/sec^2 C 7.06×10^4 ft/sec^2
B 7.06×10^3 ft/sec^2 D 7.06×10^5 ft/sec^2
E none of these

19. Use the equation $v^2 = 2ad$ to find the acceleration. Select your answer from the choices given in Question 18.

Please return to page 95 in the STUDY GUIDE.

AMP LESSON 6 PROBLEM ASSIGNMENT

Articulated Multimedia Physics

LESSON 6

1. In a 4-hour period, a man walked 2.5 mi during the first hour and 2.2 mi during the second hour. He then rested for an hour, after which he walked 2.3 mi during the fourth hour. What was his average speed during the entire 4 hour period?

2. A ball rolls down a hill with a constant acceleration of 3 m/sec^2. If it starts from rest,

    (a) what is its speed at the end of 4 sec;
    (b) how far did it move?

3. A car moving on a straight road increases its speed at a uniform rate from 20 ft/sec to 30 ft/sec in 20 sec.

    (a) What is the acceleration of the car?
    (b) How far did it move during the 20 sec interval?

4. A ball thrown vertically upward returns to the ground 6 sec later.

    (a) For how many seconds did the ball fall after attaining maximum height?
    (b) How high did it go?
    (c) With what velocity did it strike the ground?

5. A ball rolls down an incline 12 m long in 3 sec. Assuming that the acceleration is uniform,

    (a) what was the average velocity of the ball;
    (b) what is its final velocity?

6. A car traveling at 25 m/sec is brought to rest at a constant rate in 20 sec by its brakes.

    (a) What was its acceleration?
    (b) How far did it move after the brake was applied?

7. A bullet leaves the muzzle of a gun at a speed of 400 m/sec. The length of the gun barrel is 0.5 m. Assuming uniform acceleration,

    (a) what was the average speed of the bullet inside the barrel;
    (b) how long was the bullet in the barrel after the gun was fired? (Use scientific notation).

8. Pressing the brake slows a car down from a speed of 90 ft/sec to 50 ft/sec in 8 sec. Assuming the acceleration to be uniform, how far did the car travel during the 8 sec interval?

(next page , please)

9. An electron is accelerated uniformly from rest to a speed of 2.0 x 107 m/sec.

   (a) If the electron traveled 10 cm while it was being accelerated, what was its acceleration?

   (b) How long did it take it to acquire its final speed? (Use scientific notation for both answers.)

10. During a 30 sec interval, the speed of a rocket rose from 200 m/sec to 500 m/sec. How far did the rocket travel during this time?